CfBT Education Trust

Enhancing primary mathematics teaching and learning

Research report

Edited by Professor David Burghes

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Welcome to the Centre for Innovation in Mathematics Teaching (CIMT)

CIMT is a self-financing centre in the Faculty of Education at Plymouth University. It was set up some 25 years ago, initially at the University of Exeter, with a research and development focus, aiming to support and help teachers of mathematics to implement good practice, based on international work.

It moved to the University of Plymouth in July 2005, and has since been joined at the Faculty of Education by the Royal Statistical Society's Centre for Statistical Education. The two centres are co-located to provide a thriving, innovative and enterprising facility for pedagogical research and development in the mathematical sciences.

In the past two decades CIMT has undertaken two major international longitudinal studies, namely the Kassel project (mathematical progress in cohorts of pupils in 15 countries in their last three years of compulsory education) and the IPMA project (mathematical progress of pupils in the first five or six years of school) both aiming to make recommendations for good practice in mathematics teaching and learning. The dissemination phase for UK schools of both of these projects is through the Mathematics Enhancement Programme (MEP), the resources all being freely available at the CIMT website: http://www.cimt.plymouth.ac.uk.

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About the authors

David Burghes

David has been Professor of Mathematics Education at Plymouth University since 2005. He was previously Professor of Education at the University of Exeter, where he founded the Centre for Innovation in Mathematics Teaching (CIMT). The Centre was set up to provide help and support to mathematics teachers around the world through research, development, evaluation and dissemination of good practice in mathematics teaching and learning.

david.burghes@plymouth.ac.uk

Tibor Szalontai

Tibor works at the Institute of Mathematics and Informatics of the University College of Nyíregyháza, Hungary, where he has been a tutor of mathematical and methodological subjects since 1984.

szalonta@zeus.nyf.hu

Masataka Koyama

Masataka is Professor of Mathematics Education and a Vice-Dean of the Graduate School of Education at Hiroshima University in Japan. His research interests include students' mathematical understanding, teachers' professional development and school mathematics curriculum and textbooks.

mkoyama@hiroshima-u.ac.jp

Sirkku Myllyntausta

Sirkku is Head of Mathematics at Helsinki University Viikki Teacher Training School, Finland. She has been at the forefront of the enhancement of mathematics education in Finland over the past two decades and is a mentor teacher for student teachers at the University Practice School.

sirkku.myllyntausta@helsinki.fi

Yvonne Beston

Yvonne is a Sessional Lecturer in the Education Department at London Southbank University. She was previously headteacher of a large primary school in Orpington, Kent, where she took the lead in implementing the Mathematics Enhancement Programme in her school.

y.beston@btinternet.com

Melanie Hazell

Melanie teaches at Long Lane Primary School, Reading, Berkshire. She was a supporter of the Mathematics Enhancement Programme; video clips from one of her lessons are on the CIMT website.

office.longlane@westberks.org

Derek Robinson

Derek is Head of Mathematics at Bishop Luffa School, a high-performing mixed comprehensive school in Chichester, West Sussex. He has worked with CIMT at Plymouth University for the past decade and has been instrumental in advocating and practising Japanese lesson study.

djrjpr@aol.com

Pam Pitman

Pam is Headteacher at Grouville School in Jersey (a British crown dependency and the largest of the Channel Islands). The school is two-form entry, with Nursery and Reception classes. Pam has encouraged and supported the initiative to implement the Mathematics Enhancement Programme at her school.

p.pitman@grouville.sch.je

Rachel Smith

Rachel is Mathematics Co-ordinator at Grouville School, Jersey. She was instrumental in the introduction of the Mathematics Enhancement Programme and lesson study in her school, responsible for co-ordination and leadership of the project.

r.smith@grouville.sch.je

Sharon Kellett

Sharon is Nursery Teacher and Foundation Stage Co-ordinator at Grouville School, Jersey. Sharon has flourished in her role of enhancing mathematics in the Nursery at her school.

admin@grouville.sch.je

Jodie Hunter

Jodie is a lecturer in Mathematics Education at Massey University, New Zealand but was previously based in CIMT at Plymouth University. She has been working with primary schools in England and Jersey, helping them to enhance their mathematics provision.

jodie.hunter@plymouth.ac.uk

Acknowledgements

One of the keys to enhanced progress in mathematics for young people is a sound mathematical foundation at primary level, delivered with enthusiasm, confidence and flair.

The international contributors to this publication, Tibor Szalontai, Masataka Koyama and Sirkku Myllyntausta, write from the perspective of high-performing countries in worldwide surveys of mathematical attainment, making the descriptions of practices in their countries especially relevant to those working to enhance standards in their own situation. Sections giving accounts of implementation of changes to practice in schools in England and Jersey are of particular interest as they are written by the teachers involved, rather than outside observers. We are grateful to these teachers for giving an insight into their experiences.

Although it has been written with the aim of improving practice in the UK, we hope that this publication will also be of interest and help to those involved in mathematics education in other countries.

We are grateful to all the contributors and their colleagues whose work has enabled the Centre for Innovation in Mathematics Teaching (CIMT) to compile this publication. In particular, we are grateful to Jodie Hunter who undertook the initial editing of many of the contributions, and to our funders, CfBT Education Trust, for their continued support of initiatives vital to the enhancement of education, both in the UK and worldwide.

Introduction

Professor David Burghes, *CIMT, Faculty of Health, Education and Society, Plymouth University*

In the 21st century, an important goal of education is to develop individuals with a high level of mathematical proficiency which then supports future participation in employment and citizenship. Mathematical knowledge is fundamental to the understanding and development of science and technology as well as being applicable to many areas in the social sciences. It is vitally important for all countries in this highly competitive global economic environment, yet there are continued difficulties in developing a successful education system which supports all pupils to reach their mathematical potential.

There are many reasons for this in the UK (and many other countries with developed economies); these include:

- insufficient numbers of quality teachers at secondary level
- too few primary teachers with adequate mathematical knowledge
- too much emphasis on testing rather than teaching
- a very transient teaching workforce compared to other countries.

Some of these issues are complex and not easy to change even in the medium or long term. UK governments have responded with many short-term initiatives, some of which may result in immediate gains but which are not sustainable over a longer period, neither do they solve the fundamental problems.

Here we outline a sustainable method of enhancing the teaching and learning of mathematics in the **primary** phase of education. It is based on our experiences of implementing the 'Mathematics Enhancement Programme' (MEP) in primary schools in the UK but the recommendations for enhancement of primary mathematics are suitable for any implementation and change in the classroom and not just for MEP.

Our focus on enhancing primary mathematics is based on the hypothesis that the only real way to produce young people who are mathematically confident and capable is to start in the primary phase. This is supported by consideration of mathematically high-performing countries such as, for example, Japan, Singapore, Russia, Hungary or Finland.

All these countries put in place a strong mathematical foundation in the primary sector, taught by mathematically competent teachers and enjoyed by the pupils. In countries which provide such a foundation in mathematics, students are more able to achieve at secondary level. This results in capable and creative young people entering the teaching profession, reinforcing this model of mathematical progress rather than the reverse.

Background to UK initiatives

We will first recap on the major initiatives that have been taken over the past few decades to improve primary mathematics teaching in the UK.

The 1960s and 1970s saw a strong move towards child-centred education in which children were encouraged to work at their own pace. In primary schools, formal classrooms with pupils sitting in pairs at desks almost disappeared in favour of a more child-friendly atmosphere in which children sat around tables in groups of four or six. Pupils were grouped according to ability, particularly for mathematics, with the lowest-ability groups nearest the teacher so that they could receive more support. There was also space in most classrooms for children to sit on a carpet on the floor for some of their activities.

This type of environment was particularly popular in the first four years of primary school. Not only was differentiation achieved by the group seating, it was very much encouraged by the differentiated work set for the groups. This occurred from Year 1 onwards, with the result that mathematics was rarely taught to the whole class but through activities, worksheets and practice books given to the groups.

The result, of course, was a wide attainment spread when these pupils entered secondary schools at age 11+ after their six years in primary. Cockcroft, in the 1982 report 'Mathematics counts'[1] refers to the '7 year gap' in the spread of attainment on entry to secondary schools.

One of the main recommendations made by Cockcroft was to encourage a more investigative approach to mathematics. This was well received by most primary teachers: it fitted in well with the group activities which dominated much of the early years, and resulted in many more investigational activities being used.

The 1980s saw a strong move to teach primary mathematics through understanding rather than by 'rote learning': projects such as the CAN (Calculator Awareness Number) Project [2a, 2b] discouraged the learning of multiplication tables but allowed pupils unlimited access to simple calculators.

The result of all these trends appeared to be a lack of basic numeracy in the younger generations. Several critical articles appeared in newspapers after the results of national and international studies were reported in the early 1990s.

To counter the impression that primary schools were not sufficiently focused on learning (there was criticism not only of numeracy but also of standards in literacy), the early 1990s saw a number of complementary government initiatives. These included:

- national tests in mathematics and English at the end of Year 2 (age 7 years) and at the end of Year 6 (age 11), which became known as Key Stage 1 and Key Stage 2 tests
- national reporting of results in terms of national curriculum levels, particularly at Key Stage 2, where regional league tables were published in the national and local press
- a target set by the Government of 75% of all pupils achieving Level 4 or higher by the year 2002
- pilot numeracy and literacy projects aimed at improving performance.

These pilot projects very quickly became national initiatives with the implementation of the Government's ***National Numeracy Strategy (NNS)*** which recommended:

- a daily numeracy lesson (45 minutes for Years 1 and 2, 60 minutes for Years 3, 4, 5 and 6)
- emphasis on mental skills
- a three-part lesson structure consisting of:
  - oral and mental starters (whole-class interactive teaching)
  - a main activity (often with differentiated group work)
  - a plenary session (whole-class interactive teaching).

To help teachers implement the strategy, about 500 numeracy consultants were appointed by the local education authorities, cascade training was given and the Department for Education and Skills published its ***Framework for Teaching Mathematics***[3] which gave a very detailed structure, including weekly objectives for lessons.

After this initial boost, there was some evidence to show that there had been improvement in Key Stage test results (taken at the end of Year 2 and Year 6) but doubt was cast on whether this was real progress rather than teachers becoming more practised at 'teaching to the test' (see Cambridge Primary Review Research Briefing on Curriculum and Assessment[4]).

There did though seem to be agreement that whilst the National Numeracy Strategy had encouraged mental numeracy skills, there had been less progress in enhancing mathematical skills. In response to this, the Government asked Sir Peter Williams to chair an Inquiry into Primary Mathematics. This resulted in a report published in June 2008.[5] There were many recommendations but the main outcome has been the training of specialist mathematics teachers, through CPD (continuing professional development), with the aim of there being a specialist mathematics teacher in every primary school. The initiative is currently ongoing.

Background to CIMT's initiatives

The Centre for Innovation in Mathematics Teaching (CIMT) is a self-financing research and development centre based at the University of Plymouth, aiming to help and support teachers to enhance their teaching and pupils'/students' learning of mathematics. We have undertaken international comparative studies to help our understanding of what constitutes good practice in teaching and training; we use the model:

observe research develop implement evaluate disseminate

in our developments. Our first encounter with primary mathematics was when observing secondary mathematics teaching in Hungary in the late 1990s as part of the Kassel Project[6] (research into the teaching and learning of mathematics in secondary schools in different countries, culminating in recommendations about good practice in helping pupils achieve their mathematical potential). Having taken the opportunity to look at primary classes in this 'general school' (a combined primary and secondary school), we were amazed at the mathematical level of a Grade 4 (age 9) class. (Children in Hungary start schools at age 6 in Grade 1 and have four years in the primary sector.) This was, in fact, an accelerated class, but nevertheless, we were astonished at the mathematical level exhibited by the pupils. The teaching strategies were radically different from those observed in UK primary classrooms.

The teaching was very interactive with pupils regularly demonstrating and articulating their solutions to the rest of the class at the board. There were eight or nine related activities in the 45-minute lesson; each activity was well prepared and the teacher ensured that all pupils were involved and 'on-task' throughout the lesson. We were so impressed that, after further observations and discussions, we decided to develop and implement a similar scheme for primary mathematics in the UK, using Hungarian resources as a basis for the project.

At about this time, November 1997, the UK Government set up the Numeracy Task Force to address its concern about low levels of numeracy and essential basic skills. Many of the teaching strategies we had seen in Hungary were included but, although David Burghes, the Director of CIMT, was on the Task Force Committee, compromises were made and the strategy model of a three-part lesson became (and in many schools, continues to be) the norm in UK mathematics classrooms, despite there being no research evidence for its effectiveness.

At CIMT we continued over the following eight years to develop the **Mathematics Enhancement Programme (MEP) Primary Demonstration Project** and have worked through that time and for the past five or six years with schools keen to implement this type of course with its emphasis on providing a strong mathematical foundation. It is important at this stage to clarify that the teaching style we were recommending:

- puts the teacher as the focus of the learning
- encourages correct, precise, orderly spoken and written mathematics
- places greater emphasis on whole-class, interactive teaching with less differentiation and less individual work
- promotes mental mathematics, including knowledge of basic addition and multiplication facts.

However, it is also important to recognise that we were trying to find a model of 'great' practice but, on reflection, noting that there can be more than one model that achieves this.

The MEP Primary course is comprehensive; it can be freely accessed at:
http://www.cimt.plymouth.ac.uk/projects/mepres/primary/default.htm

For each year there are:

- two practice books (e.g. Y1a, Y1b, etc) with 175 pages in total
- copy masters provided for important diagrams
- detailed lesson plans for four of every five pages of the practice books.

We also encourage the use of:

- number lines (class and individual)
- number cards
- shape cards.

Project schools are provided with a DVD of teaching in three Hungarian classes (Reception, Year 1 and Year 8) with subtitles in English. A more comprehensive list of the key features of the model of good practice being implemented is provided below (see page 10).

Lesson study (or collaborative practice) is a common component of teaching in high-performing countries (see Chapter 1). We actively encourage this method of CPD and have found that schools and teachers are extremely enthusiastic about the benefits to their teaching that it provides. The importance of an external consultant, able to offer expertise and advice, whilst allowing teachers to develop their own strategies for their particular situations, has been emphasised throughout this publication.

Chapter 1 of this publication gives further information on good practice in primary mathematics teaching in Hungary, Japan and Finland which has been instrumental in our initiatives to improve primary mathematics teaching and learning. This is followed by details of our progress in implementing MEP Primary in the UK where we have asked the teachers involved to give an honest and frank account of the progress made (Chapters 2 to 5). We particularly focus on our implementation of MEP Primary at Grouville School in Jersey (a large non-selective state school with full ability range of pupils) in Chapter 5. Finally in Chapters 6 and 7 the team from CIMT review our implementation strategies, particularly the role of external support, making recommendations for enhancing and sustaining progress in primary mathematics teaching and learning.

Although it has been written specifically for UK primary schools, we hope this will be of interest and help to any primary teachers interested in the experiences and developments of others.

The Mathematics Enhancement Programme (MEP): key points

1. **Lesson** well prepared (teacher knows the lesson plan well and is aware of any problems/ difficulties which might occur), resources are at hand, board prepared in advance, pupils have own resources on desk.

2. **Seating** – every pupil has direct eye contact with the teacher and can get to the board quickly and easily. Able children seated beside less able.

3. **Whole-class interactive teaching** predominates, with planned intervals of individual and paired work. All pupils on task and all given the chance to demonstrate, answer, explain, suggest, criticise, etc.

4. **Friendly, non-confrontational atmosphere** where pupils learn from and support others and have fun! Mistakes used as teaching points. Encouragement given to pupils who have difficulty and praise given when deserved. Pupils are encouraged to appreciate the good work of others.

5. **Spiral curriculum** with continual revision; learning by heart encouraged with progression in small, logical steps.

6. **Visualisation and manipulatives** are used in the early years and with less able pupils. Contexts are related to pupils' experiences where possible. Demonstrating on a number line and modelling are used to help understanding.

7. **Exercises reviewed interactively** with the whole class at the same time. Pupils give the solutions, not the teacher, and rest of the class agrees/disagrees or suggests alternative solutions. Pupils are expected to correct their own work (i.e. cross out wrong answer and write correct answer in red). Teacher gives hints only if the whole class is stuck.

8. **Challenges or extension work** set for able pupils, or they help their less able neighbours; no one is inactive.

9. **Correct notation, layout and language** used at all times. Teacher acts as a model for pupils to follow (on board and orally), repeating/showing a pupil's explanation more clearly and succinctly where necessary. New words always explained and written on the board for pupils to copy in exercise books.

10. **Fast paced and varied** activities related to the concept being taught. Time limits set for individual/paired work. Time allowed for pupils to explain and for whole-class discussion.

1 Examples of good practice in mathematics teaching and learning

1.1 Hungary

Professor Tibor Szalontai, *College of Nyíregyháza, Hungary*

Outline of primary education in Hungary

The Hungarian schooling system begins prior to entering primary school with a compulsory preparatory year (for 5+ year old children) at kindergarten. Kindergarten education is a well-structured and complex process, with most children in mixed age-group classes. The activity is largely based on children's quasi-voluntary activities; however, younger children may join organised activities. Daily activity incorporates natural learning opportunities such as playing in the courtyard and walking on the street (to a city centre or a market, to a park or wood, collecting leaves or corn). Children are encouraged to paint and draw, manipulate or play with toys at almost any time, alone or with classmates. Additionally they engage in physical exercise and learn songs and verses which are frequently combined with elements of folk dance formations. There are also intensive 30-minute sessions each day which cover basic curriculum areas. This includes a session with an emphasis on mathematics once or twice a week. The focus of these sessions is on learning to count and calculate mentally (up to 10) rather than on writing and reading letters, words and numbers. For example, the children may be asked to 'draw three houses', 'put four white rabbits above the blue line' or 'add two chestnuts to make nine in the box'. Other areas such as shapes, symmetry, sequences and patterns are also covered. The emphasis in these sessions is on concentration, manipulation skills, memory, rhythm, orientations and relations.

Within Hungary, primary school begins officially for children from six years old and consists of an eight-year general school split into a lower section (Years 1–4) and upper section (Years 5–8). It is important to note that many children (approximately half) enter primary school at age seven due to either parental request or recommendation of kindergarten teachers or educational psychologists. When children begin primary school there are often notable differences in their starting achievement levels. Commonly, larger city schools attract or select higher-achieving pupils which creates competition among the schools for pupils. Consequently, there is a developing gap between stronger and more poorly-performing schools.

In Hungary, different models are used for the structure of the school day and between the lower and upper sections of the school. Within the lower section, pupils are taught by all-subject primary teachers and generally stay in school in the afternoon, during which time they practise and work on homework for 90 minutes, along with other play and cultural activities. A class might have an all-subject class teacher and an afternoon teacher. In other schools, two teachers might share the morning subjects and alternately teach the afternoon class. A further model used is of two teachers sharing the subjects in their two classes with both classes having different afternoon teachers. In contrast, in the upper section of primary school, beginning at Year 5, pupils are taught by subject teachers. Often Year 5 pupils have difficulties with the transition due to the need to adjust to new and different styles, pace of work and requirements. Within the upper section of the school only a minority of pupils stay in school for the afternoons. A small percentage of pupils leave the general school after Year 4 for an eight-year *gymnasium*, similar to a grammar school in the UK.

An important feature of both kindergarten teacher training and primary teacher training is a four-year complex course with various pedagogical, psychological, all-subjects and individual subject methodological studies. School observations and the first stages of teaching practice are carried out in university practice schools. This is followed by a placement within a non-university practice school and completed with an examination of teaching in the practice school again. As a precondition for entry to university, all the trainees are required to complete mathematics courses up to age 18 and sit the final examination. Overall, approximately two thirds of each year's cohort completes this requirement with 90% completing an intermediate level, without any differential and integral calculus.

Mathematical primary attainment in Hungary

Results from the International Project on Mathematical Attainment (IPMA, 1998-2004) showed an even and relatively strong progress of the Hungarian Year 1–5 sample. Our initial starting score in Test 1 was fifth from eight while the second test score was third, behind Singapore and China. In subsequent tests, the Hungarian results were either first or second behind China or Singapore. Our sample had progress levels similar to China and Singapore until the age of nine, but then our progress became slightly slower.[1] These results correspond with the Kassel Project (1993-1997) results where Year 9–10 Singaporean pupils scored much higher than Hungarian pupils. These results indicate that the progress of Hungarian pupils gradually lags behind the achievement of pupils from the strongest countries during the secondary years until the age of 16.[2] Similarly, results from the Trends in International Mathematics and Science Study (TIMSS) demonstrate the strong mathematical achievement of Hungarian 10-year-old pupils. However, national monitoring[3] showed a continuing decline in the secondary school mathematics attainment for decades until the turn of the millennium.

Features of primary mathematics teaching

In the Hungarian schooling system there is a smooth transition between kindergarten and Year 1 mathematics. In the first month of Year 1, children continue the focus on mental progress with mental counting and calculation, relations and orientations. As a preparation for digit and sign writing, they practise vertical, horizontal then slanting line patterns in square grid rows followed by curved line patterns. The grid size is reduced gradually to the normal size. While children work on this, the teacher monitors, corrects and evaluates (praises) the work of each child at any time.

In the autumn term, children are introduced to the reading and writing of number digits in a very systematic way from 1 to 10 in their order, followed by numbers up to 20 in the spring term. Many different models of the number are connected to the digit either in the textbooks or in the classroom. Decomposition, completing, taking away and addition, comparison, relations and ordering are always modelled with pictures or drawings or manipulative tools such as corn pieces, coins and sticks. The operations of addition and subtraction are always introduced at the same time, along with open sentences (equations and inequalities), relations and ordering. These are frequently built in real life or fantasy contexts. When pupils have become skilled at reading and writing numbers, they are introduced to representations on number lines.

During the first two years of schooling, many teachers prepare colourful demonstrative models for the questions which they have developed or use from the textbooks. These are used for whole-class interactive work or for discussion after the individual work. The textbooks used include carrying practice along with challenging problems and puzzles. As children progress into Year 3 the presentation of the mathematics lessons becomes less colourful and there is a greater emphasis on performance and competition. There is little use of ICT in Hungarian primary mathematics lessons aside from the use of overhead projectors.

The mathematics curriculum within Hungary includes the introduction of mental multiplication and division from Year 2 and the four operations involving numbers up to 100 also in this year. From Year 3, formal addition, subtraction and multiplication are taught, and formal division is introduced from Year 4. In contrast to the mathematics curriculum in other countries, Hungarian primary pupils are taught about decimals and percentages only from Year 5 and the use of rectangle or circle fraction models is frequent. They meet with negative numbers rarely until Year 4. Additionally data handling and probability are not taught prior to Year 5 and pictograms are rarely used. Often long text word problems pose difficulties for many Hungarian pupils as more typically they meet shorter context questions. For example, Hungarian pupils are more experienced in combinatorial questions.

Key overall findings from Hungarian research[6, 7]

- At the start of schooling there is variation in the attainment of pupils from school to school and within classes. This is due to a number of factors including the non-formal preparation of pupils in kindergarten and varied home-based learning experiences during their early years. A secondary reason is that stronger city schools can attract or select higher-achieving pupils than schools in small villages.
- Gender differences in mathematics attainment are not significant for pupils in Years 1–6.
- Attainment levels within rural schools are much lower than in city schools. The capital city of Hungary has the strongest achievement results, followed by the larger cities and smaller cities and then deprived area villages. An important factor in pupil achievement is the parents' educational level and social background. Reasons for poor performance frequently cited by teachers who work within deprived areas are: less able pupils; deprived home backgrounds; lack of parental support; poor attention; passivity and discipline and behaviour problems. For example, in small rural schools many pupils are not supported by different learning experiences at home and additionally do not remain in school for the afternoon.
- From Year 2 in stronger performing and larger schools, results from testing within class cohorts have less spread. This may be attributed to frequent use of whole-class interactive teaching and the uniform primary teacher training provided.
- Hungarian primary teachers are generally women and the majority of Year 5–8 mathematics (two-subject) teachers are women too.
- The role of the teacher and school is crucial in pupil progress as the best teachers are able to motivate, educate and improve understanding for children from a range of backgrounds.
- Teachers prepare detailed lesson plans for each lesson, based on the identified objectives for year levels.
- Homework is a consistent feature during weekdays: however, it is not marked, but discussed in the next lesson.
- Classrooms are arranged as neither too homogenous nor too heterogeneous, and these mixed-attainment classes are used to optimise the progress of pupils.

Key findings about effective teaching practice in Hungary[8]

- A range of teaching styles can lead to effective learning of mathematics. These may include: a focus on precise, disciplined work; developing good relationships with pupils; looking at individual progress; providing challenging problems; teaching in a calm, unhurried manner.
- The most frequent classroom practice is whole-class interactive teaching with common activities and periods of individual work followed by whole-class (group) discussion.
- The national curriculum and Hungarian textbooks prescribe a spiralling and concentric widening of knowledge and abilities or skills.
- Rich and varied models are adopted and further developed by expert teachers.
- When applicable, learning starts with pupils experimenting with manipulative tools.
- There is a need to find a balance between numeracy, notation, invented strategies, knowledge, abstract reasoning, symbolism and practical, everyday knowledge – and the skills and problem-solving abilities required for everyday life.
- Effective lessons include a balance of work and, across schooling levels, there should be a gradually rising level of workload. Pupils should be provided with written and spoken expectations and the level of difficulty and requirements of the work should be consciously developed.
- It is important to create a positive learning environment with a focus on disciplined work. The effective teacher maintains control and keeps the pupils on task through positive motivation and the use of praise.
- At times, differentiated level group work, and sometimes heterogeneous group work and pair work, can be seen in the lessons. However, the majority of lessons incorporate a naturally differentiating method as this is more effective: in addition to the standard whole-class questions and activities, the teacher sets some extra puzzles which are based on logical reasoning rather than calculation. Pupils who have already solved the standard question or task before discussion may complete these problems. Solutions and key ideas are discussed at the end of the lesson or next time. Alternatively, differentiation may occur with the expectation that pupils complete more or fewer sub-questions within a particular timeframe.

- In the most effective lessons, teachers set between four and eight questions for individual work (usually after a common, mental introduction) which are then discussed. The main stages of this work are as follows:
  1. **Set the question or task,** and start the work.
  2. **Monitor the individual work.** (The teacher may help lower-achieving pupils.)
  3. **Stop the work** and begin discussion.
  4. **Pupils report on their work.** (Followed by feedback: 'Who agrees?', 'Who wrote this?', 'Who thought something else?')
  5. **Reasoning** (debate).
  6. **Agreement on the correct plans and solution.** (The teacher confirms these this time.)
  7. **Feedback on success.** ('Who wrote a correct plan?', 'Who got the correct result?')
  8. **Self-correction by the pupils.** ('Cross through the incorrect plan/result with red.' 'Write the correct result.')
  9. **Evaluation, praise, encouragement.**
  10. **Other.** For example, teacher's proof, whole-class expansion, generalisation, historical data and applications.

In accordance with the type of lesson or with the level of skills, the weight of these actions might be varied. Both spoken and written abilities and skills can be developed in lessons of this type.

Finally, we would like to stress that these examples of effective teaching were found in a limited sample of schools and so we cannot state that all of them are widespread in Hungarian primary mathematics education.

1.2 Japan

Professor Masataka Koyama, *Hiroshima University, Japan*

Background

We first note some fundamental information needed to understand good practice in mathematics teaching and learning in Japan (Koyama[9, 10, 11, 12]; Ministry of Education[13, 14]).

- The school education system comprises six-year primary school, three-year lower secondary school, and three-year upper secondary school. The first two levels – altogether nine years, from age 6 to age 15 – are compulsory for all children.
- The school year begins on 1 April and ends on 31 March of the following year.
- The national curriculum standard is prescribed in the 'Course of Study' determined and issued by the Ministry of Education, Culture, Sports, Science and Technology *(Monbukagakusho)*. The Course of Study provides the basic framework for the mathematics curriculum; the required time spent on mathematics, the overall aim of mathematics, and the objectives and content of mathematics teaching and learning at each grade in school.
- School textbooks must be approved by the Ministry of Education according to the Course of Study.
- Teaching practice in pre-service teacher training is mainly undertaken by university-attached schools (university practice schools).
- Public (i.e. state) school teachers are local prefectural or municipal public officials and are appointed by the respective local prefectural or municipal boards of education in which the schools are located.
- During their first year of teaching, as part of in-service teacher training, new teachers receive induction training, and are then formally appointed if they gain a favourable evaluation of their performance during this conditional employment period.
- The induction training programme is spread out over the school year for a total of at least 90 days. Of this, 60 days or more are school-based, during which the new teachers receive instruction from their advising teacher, and not less than 30 days are spent attending lectures, seminars and various practical training sessions that include five-day workshops held in education centres or other institutions outside the school.
- All public school teachers, including principals and vice-principals, are required to move from one school to another school several times in their careers, usually every three to ten years, within their local city or prefecture.
- Primary school teachers teach almost all school subjects at their own grade in the classroom. They are not necessary specialists in mathematics education.

A typical model of good practice

A typical model of good practice recognised by many Japanese educators and teachers is the 'problem solving lesson' in the mathematics classroom (see Becker & Shimada,[15] Stigler & Hiebert,[16] Burghes & Robinson[17]). The problem solving lesson is structured and progressed through four distinct phases: presentation of the problem; developing a solution; progress through discussion; summarising the lesson (see Burghes & Robinson[17]).

There are a number of reasons that the problem solving lesson is recognised as a good model for teaching and learning mathematics. First, an essential element of doing mathematics is the *process of solving* a problem mathematically rather than its product. If students are skilled in the process of solving problems mathematically, then they are able to apply their mathematical knowledge, skills and ways of mathematical thinking to new or unfamiliar situations. Mathematics lessons should incorporate learning activities which are structured so that students progress through the process, solving problems mathematically. The use of problem solving lessons incorporates a student-centred approach which can encourage students to construct mathematics collaboratively by using their acquired mathematical knowledge, skills and ways of mathematical thinking in the classroom.

As Sawada[18] points out, the advantages of using the problem solving lesson model for teaching and learning primary mathematics in the classroom are:

- Students participate more actively in the lesson and express their different ideas or solutions more frequently.
- Students have more opportunities to make comprehensive use of their knowledge, skills and ways of thinking.
- Even low-achieving students can respond to the problem in significant ways of their own.
- Students are instinctively motivated to give their justifications or proof.
- Students experience the pleasure of mathematical activities and receive approval from peer students in the classroom.

An example of a lesson plan

In the next section, we describe an example of a lesson plan used in a 'research lesson' in a cycle of lesson study conducted in a public primary school.[19] This lesson is structured using the problem solving lesson model in teaching the 'Multiplication in Vertical Form' unit in the 3rd grade.

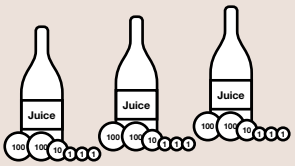
A key aim of this unit is for students to develop their understanding that multiplication can be calculated by using the following multiplication rule:

"the product of multiplication does not change even if we multiply by decomposing a multiplicand or a multiplier into some numbers."

The teacher helps students understand that it is an easier way to calculate multiplications based on the place value system of decimal notation, by using Japanese yen coins. Furthermore, the teacher emphasises that students do not have to memorise the way of multiplying in vertical form, but be able to understand and explain the meaning of multiplication through such activities as explaining with pictures and calculating separately for each position of the multiplicand.

In this lesson, students are expected to think about and work out how to calculate a multiplication such as '213 × 3' by applying a previously-learned method of multiplying a two-digit number by a one-digit number. The teacher asks students to explain their way of multiplication in vertical form by focusing on numerical positions like ones, tens, and hundreds of a multiplicand. The teacher gives students two types of exercises at the end of this lesson. The first type involves the multiplication of three-digit numbers by one-digit numbers in vertical form. In doing this exercise, students work in pairs so they are provided with opportunities to explain their ideas to each other and analyse the meaning of multiplication and their solution strategies. The second type of exercise involves a missing-value problem. Similarly, students work in pairs to solve the problems and explain their strategies to deepen their understanding of the content of this lesson.

Phase 1: Presentation of the problem

	Learning activities and main questions	Students' thinking and responses	☐: Remarks ☆: Evaluations
Understanding a task and making a plan (10 minutes)	**1. Understanding a task** ■ Let's write the problem in your notebook. **I bought three bottles of juice. The price of each bottle is 213 yen. How much is the total payment?** ■ Let's write this problem as a mathematical expression. ■ Let's think about this problem by using pictures of Japanese yen coins. ■ Let's write the objectives of this lesson in your notebook. **Let's think about the ways of multiplying a three-digit number by a one-digit number.**	• I may find the answer to this problem by multiplication. • The expression of this problem is '213 × 3' because the price of each bottle is 213 yen and I bought three bottles. • I guess today's topic may be multiplication of a three-digit number by a one-digit number.  • The total payment is 639 yen. • I may find the total payment by counting. • I may find the total payment by multiplying.	☐ Teacher will reconfirm that this problem is about the situation of multiplication. Then he/she will construct a mathematical expression for this problem with the students and clarify any difference from the problems that students have already learned. ☐ Teacher will ask students to think about this problem by using pictures of Japanese yen coins on a blackboard.

Phase 2: Developing a solution

	Learning activities and main questions	Students' thinking and responses	☐: Remarks ☆: Evaluations
Thinking about the problem and deepening their ideas (30 minutes)	**2. Making a plan to solve the problem** ■ How can we calculate this? **(What students have already learned):** Students are expected to recall the method of multiplying a two-digit number by a one-digit number.	• I will think about this calculation by decomposing 213 yen into 200 yen, 10 yen and 3 yen. • I will try to multiply separately for each position. • I will calculate it in vertical form.	☐ Teacher will ask a few of the students who have their own ideas to suggest a hint if others are finding the problem difficult.
	3. Solving the problem for themselves individually ■ Let's think about the method of calculating and write your own method in your notebook. **(Mathematical activity):** Students are expected to write their method of multiplying a three-digit number by a one-digit number in their notebook, by considering the meaning of calculation in each position.	**1.** Calculating the multiplication separately for each position of the multiplicand 639 → 3 × 3 = 9; 10 × 3 = 30; 200 × 3 = 600 **Total 639** **2.** Vertical form 213 × 3 9 (3 × 3 = 9) 30 (10 × 3 = 30) 600 (200 × 3 = 600) **639** *(Error:)* 213 × 3 9 3 6 **18**	☆ Students are able to think about and explain how to multiply a three-digit number by a one-digit number, by using the method of multiplying a two-digit number by a one-digit number. *<Mathematical Thinking>*

Phase 3: Progress through discussion

	Learning activities and main questions	Students' thinking and responses	☐: Remarks ☆: Evaluations
Thinking about the problem and deepening their ideas (30 minutes)	**4. Solving the problem in a small group or a pair** ■ Let's explain your way of calculating to others. **(Representation):** Students are expected to explain the method they use, being aware of other students in the class and talking clearly about the rationale of their method.	**1.** I calculate the multiplication separately for each position of the multiplicand as follows: 3 × 3 = 9 — 9 yen 0 × 3 = 30 — 30 yen 200 × 3 = 600 — 600 yen **The total is 639 yen.**	☐ Teacher will ask students to explain the meaning of the calculation in each position of the multiplicand with their expressions and pictures.
	■ Let's understand the way of multiplying in vertical form.	**2.** I calculate in vertical form as follows: • Multiply the multiplier by the ones, tens and hundreds respectively. • Write 9 in the ones position because 3 × 3 = 9. Write 3 in the tens position because 10 × 3 = 30. Write 6 in the hundreds position because 200 × 3 = 600.	☐ Teacher will pick up a student's error first and ask students to think about why it is not correct. ☐ Teacher will introduce the abbreviated notation of the vertical form. ☐ Teacher will explain that it is easy to multiply from the multiplier to the multiplicand.

	Learning activities and main questions	Students' thinking and responses	☐: Remarks ☆: Evaluations
Thinking about the problem and deepening their ideas (30 minutes)	**5. Exercises** ■ Let's multiply in vertical form 1 142 × 2 2 423 × 2 3 312 × 3	1 142 × 2 = 284 2 423 × 2 = 846 3 312 × 3 = 936	☆ Students understand the way of multiplying a three-digit number by a one-digit number in vertical form. *<Knowledge and Understanding>*
	■ Let's explain to each other in pairs how to multiply in vertical form. (Mathematical thinking): Students are expected to think about the method and its meaning of multiplying in the vertical form. ■ Let's explain how to multiply in vertical form to others in the class.	**1.** Write 4 in the ones position because 2 × 2 = 4. Write 8 in the tens position because 40 × 2 = 80. Write 2 in the hundreds position because 100 × 2 = 200. **2.** Write 6 in the ones position because 3 × 2 = 6. Write 4 in the tens position because 20 × 2 = 40. Write 8 in the hundreds position because 400 × 2 = 800. **3.** Write 6 in the ones position because 2 × 3 = 6. Write 3 in the tens position because 10 × 3 = 30. Write 9 in the hundreds position because 300 × 3 = 900.	☐ Students confirm the method of multiplying in vertical form by explaining their own idea to other members of the class and listening to the ideas of others.
	■ Let's fill in the blanks. (Worksheet) ☐☐☐ × 3 = 963 ■ Let's think about this problem in pairs. ■ Let's explain to others how to solve it.	• The blank ☐ in the ones position is 1 because 3 × ☐ = 3. The blank ☐ in the tens position is 2 because 3 × ☐ = 6. The blank ☐ in the hundreds position is 3 because 3 × ☐ = 9.	☐ Teacher will ask students to solve the problem. If necessary, teacher will ask them to suggest hints to each other.

Phase 4: Summarising the lesson

	Learning activities and main questions	Students' thinking and responses	☐: Remarks ☆: Evaluations
Summarising (5 minutes)	6. Summarising the lesson ■ Let's write what you have learned in this lesson in your notebook.	**1.** I have learned that we can calculate multiplying a three-digit number by a one-digit number in the same way as calculating separately for each position of the multiplicand. **2.** I understand that the multiplication in the hundreds position means the number of one hundred.	☐ Teacher will summarise the learning that has taken place in this lesson, using students' ideas, and write the summary on the blackboard.
	■ Let's write your reflection on your learning in this lesson in your notebook.		☐ Teacher will ask students to reflect on their learning and to self-evaluate.

Self-evaluation by students: Tick one symbol in each section.

1	I could understand the content of this lesson.	◎ ○ △
2	I could make use of ideas that I had already learned.	◎ ○ △
3	I could explain my own ideas to other students.	◎ ○ △

◎ Fully understood

○ Understood but not yet confident

△ Not too sure yet

Lesson study and 'cultural scripts' of good practice

In addition to the programmes officially designated for teacher training, the process of lesson study is part of Japanese tradition and culture for the professional development of all teachers with different teaching experience or positions. The Japanese lesson study model and the typical process of lesson study are well known internationally (Stigler & Hiebert;[16] Lewis;[20] Isoda, Stephens, Ohara & Miyakawa;[21] Burghes & Robinson;[17] Shimizu;[22] Takahashi[23]). The success of lesson study can be found in two main aspects: improvements in teacher practice and the promotion of collaboration among teachers. Lesson study provides Japanese teachers with opportunities to make sense of educational ideas within their own practice, to change their perspectives about teaching and learning, and to learn to see their practice from students' perspectives (Takahashi[23] p. 169). Chapter 4 and section 5.2 in this report discuss how successful lesson study can be when implementing changes in the UK classroom environment.

During lesson study, teachers collaborate to:

1. formulate long-term goals for student learning and development
2. plan and conduct lessons based on research and observation in order to apply these long-term goals to actual classroom practice for particular academic content
3. carefully observe the levels of students' learning, their engagement and their behaviour during the lesson
4. hold post-lesson discussions with their collaborative groups to discuss and revise the lesson accordingly (Lewis[20]).

In Japan, this lesson study model is used both in in-service teacher training and also in pre-service teacher training when student teachers do their teaching practice at a university-attached school. This particularly focuses on planning, observing and reflecting on the lesson with their advising (co-operating) teacher and peers. Corey et al.[24] (pp. 438-478) explored the conceptions and 'cultural scripts' (Stigler & Hiebert[16]) of a group of Japanese mathematics teachers by analysing the conversations between co-operating teachers and student teachers at a junior high school affiliated with a university in Southern Japan. He found six principles which he viewed as organising and summarising the conception of high-quality instruction of these teachers:

- *The intellectual engagement principle*: High-quality mathematics instruction intellectually engages students with important mathematics.
- *The goal principle*: An ideal lesson is guided by an explicit and specific set of goals that address student motivation, student performance, and student understanding.
- *The flow principle*: The flow of an ideal lesson is built from a question or a problem that students view as being problematic. As students engage intellectually with the problem, building on their previous knowledge, they are supported in learning the lesson's big mathematical ideas.
- *The unit principle*: A lesson is created in the framework of past and future lessons, particularly between lessons in a unit but also between units and grade levels. The lessons in a unit help students progress to ways of thinking, writing, and representing mathematical evidence in the discipline of mathematics.
- *The adaptive instruction principle*: High-quality instruction adapts so that all students are engaged in mathematical work that appropriately challenges their current understanding.
- *The preparation principle*: High-quality instruction requires a well-thought-out, detailed lesson plan that addresses the previous five principles and interconnects them in a coherent lesson.

(Corey et al.[24] pp. 450-461)

The flow principle suggests the problem solving lesson model for teaching and learning primary mathematics in the classroom. The example of a lesson plan for primary mathematics provided earlier in this section illustrates the principles of intellectual engagement, goal, unit, adaptive instruction and preparation. The intellectual engagement principle is very important for good practice in primary mathematics, because primary teachers are not necessarily mathematics specialists. As the unit principle suggests, for primary teachers who teach mathematics at their own grade, it is also important to consider thoughtfully the connection and coherence between lessons in a unit but also between units and grade levels.

Although these principles are from a study about secondary mathematics instruction, there are similar 'cultural scripts' and widely-shared conceptions of what constitutes good (effective or high-quality) practice in primary mathematics teaching and learning. Therefore it can be said that lesson study as a Japanese tradition and culture for continuing professional development (CPD) of teachers has been used in pre-service and in-service teacher training to build and maintain Japanese 'cultural scripts' of good practice in mathematics teaching and learning at both primary and secondary levels.

This author believes that it is not only the lesson plan itself that is important but also the process of developing it. In the process of developing a lesson plan for good practice in primary mathematics, teachers need to:

- reflect on their own philosophy and perspective of teaching and learning primary mathematics
- analyse and make clear objectives of learning a particular mathematical idea
- investigate teaching materials and problems (tasks) to be posed in the lesson
- think about different teaching methods appropriate for students and a topic to be taught in the lesson
- think about and choose how to word key questions in the lesson
- anticipate students' processes of understanding, misconceptions and behaviour in the lesson.

See Chapter 4 (page 39) for further discussion and examination of the lesson study approach.

Note: This article is partly based on work supported by a grant-in-aid for scientific research from the Japan Society for the Promotion of Science (JSPS), grant number 22530978. Any opinions, findings and conclusions or recommendations are those of the author and do not necessarily reflect the views of the JSPS.

1.3 Finland

Sirkku Myllyntausta, *Helsinki University Viikki Teacher Training School, Finland*

What is good mathematics teaching?
Key questions for teachers are:

- What makes the teaching of mathematics motivating and interesting to pupils?
- What methods are the most successful in helping pupils understand the concepts of mathematics and the relationships between them?

In this article, as a primary school teacher, the author concentrates on the teaching and the methods of mathematics in the lower comprehensive school, i.e. from grades 1 to 6 (ages 7 to 12). However, the materials and pupil-centred teaching can also be used in the upper comprehensive school, i.e. from grades 7 to 9 (ages 13 to 15).

Within differing levels of education there is an emphasis on different aspects of teaching, alongside this is the role of the teacher and their beliefs about what is important in teaching (Leino[25]; Pehkonen & Lepman[26]). Teachers' beliefs influence their methods of teaching, which in turn convey ideas about the nature of mathematics to pupils. For example, a traditional view of mathematics considers it as a hierarchical construction of knowledge where the knowledge represented is objective, faultless and universal; therefore, the main aim of teaching is to convey this knowledge. In contrast, the view of mathematics as a human activity considers the strategies people use to solve problems, the context, continuous change, cultural factors, school subjects and the practice that represents them and the commitment to values and imperfection. Alternatively, the teaching of mathematics can emphasise problem-centred teaching and the process itself (Leino[27]).

The aims of mathematics teaching in a Finnish context
The Finnish National Core Curriculum of Mathematics (2004) advocates mathematics teaching which is underpinned by the theory of constructivism. Therefore, importance is placed on the meaning of the language for both thinking about mathematical concepts and learning mathematical knowledge. Also there is recognition that pupils require experiences which involve meaningful learning processes. Teachers take an important role in identifying pupils' levels of development and providing appropriate opportunities for pupils to create their own internal models of the mathematical knowledge in question (Yrjönsuuri[28]).

The key purpose of teaching mathematics is to offer pupils possibilities to develop mathematical thinking, learn mathematical concepts and develop efficient strategies to solve problems or tasks. Effective teaching strategies develop both creative and exact thinking and guide pupils to analyse problems and find solutions. Systematic mathematics teaching forms a solid basis for pupils to develop an understanding of mathematical concepts and structures. The use of concrete instruments can support pupils to bridge their experiences and mathematical reasoning with the abstract mathematical system. There is a need to provide pupils with everyday problems which can be solved through mathematical thinking or the effective use of mathematical operations. Also information technology should be used to support pupils' learning process (The Finnish National Core Curriculum of Mathematics, 2004).

In Finland in grades 1 to 2, the main tasks in mathematics teaching are to develop mathematical thinking, to practise concentrating, listening and communicating, and to gain enough experience to be able to form the basis for the understanding of mathematical concepts and systems. In grades 3 to 5, the basic task of teaching is to develop mathematical thinking, prepare the ground for learning mathematical thinking strategies, strengthen the number concept as well as the four fundamental arithmetical operations, and gain experience in order to form the basis for understanding mathematical concepts and systems. During grades 6 to 9, the main task is to deepen the understanding of mathematical concepts and offer sufficient basic skills: modelling of everyday problems, learning the mathematical models of thinking, and practising remembering, concentrating and expressing things in a precise way. The aim is to provide every pupil with the basic mathematical knowledge and skills that form the foundation to his or her studies on the next level and provide the skills required in everyday life as well as in working life (The Finnish National Core Curriculum of Mathematics, 2004).

The modern belief of teaching – constructivism

Pupils' view of mathematics is defined as a combination of knowledge, beliefs, conceptions, attitudes and emotions that are developed with exposure to different experiences in mathematics. In addition, self-esteem and self-confidence take a central meaning in how pupils view mathematics. The perception of mathematics influences the study of mathematics and similarly pupils' experiences of mathematics can influence their success, self-esteem and self-confidence with the subject (Pietilä[29]). Therefore, in the teaching of mathematics it is important to develop a positive atmosphere and offer different kinds of experiences in mathematics, in order to help the pupil to develop a positive disposition towards the subject.

According to constructivism, knowledge is a mental representation that a person constructs in his or her mind. Within this framework, learning is the process of active construction by the learner which also requires an active contribution from the learner (Malinen and Pehkonen[30]). Therefore, within mathematics lessons, the emphasis is on teaching mathematical thinking, not only mechanical skills. More attention is paid to *how* to learn, than to *what* to learn. When developing mathematical thinking, the ideal is that pupils experience mathematics as something enjoyable. To maintain motivation in class, traditional teaching should be enriched with different kinds of pupil-centred methods: for example, using the problem-based approach, introducing more discussion into mathematics lessons, using different kinds of activities for pupils, using interactive and co-operational methods, using the versatility of project work, and emphasising the meaning of mathematics in everyday life. It is very important to provide pupils with opportunities to discuss mathematics with other pupils and the teacher, to use mathematical terms, and 'to speak mathematics'. It is also essential that pupils discover and process information themselves. The pupils need concrete materials to understand the subject matter, and they often become enthusiastic about them (Pietilä[29]).

Within the context of a primary school, teaching should include pupil-centred activities, functional and operational elements, games and playing, concrete everyday problems and situations, and mathematical explanations, particularly for younger students. This moves away from a dependency on books and instead advocates that study books should be used flexibly and selectively as support material (Leino[27]). Calculators and computers are also introduced into mathematics lessons in primary school. Pupils learn by using all their five senses, and it is the teacher's responsibility to see that every sense can be used in the learning process; for instance, the tactile or the kinetic sense. Learning using different senses also includes using different strategies. The strategies can already exist or they can be learned. A person can use different kinds of strategies or the same strategy in different ways to support the learning. It is important that the teacher is aware of the different strategies and the different ways of using them. The teacher should create situations in which the child has an opportunity to learn to use the different kinds of strategies in social interaction with adults or other children (Kajetski and Salminen[31]). This emphasises the importance of motivational classroom materials that provide opportunities for concrete operations in mathematics teaching.

How can co-operational methods help in mathematics teaching?
Mathematics is a school subject that very easily divides pupils into those who know and learn and those who have difficulties. The central belief behind the co-operational theory is that everyone can become a better learner by helping others to learn, and by learning together with others.

Learning involves creating subjective meanings by adding new things to what has been learned before. One of the main ideas of co-operation is to encourage productive talk and discussion. In this context, productive talk means telling, explaining, describing and debating about the learning objective. Effective learning also includes active listening and a will to understand what others are saying (Berry and Sahlberg[32]).

Developing productive talk in the classroom can be facilitated through carefully structuring the way in which the pupils work. One strategy for increasing productive talk is to use small and safe groups, as in these kinds of groups the silent and withdrawn pupils may more easily take part in mutual learning. The rules can be agreed on: everyone has the right to speak, a duty to listen to others and the mutual responsibility of taking part, as well as permission to ask for help if needed. If the teaching has been carefully planned, the use of heterogenic groups and the methodological solutions of co-operation help the lower-achieving and less motivated pupils. A further way to enrich mathematics teaching is to plan mathematical problems in such a way that they demand versatile talents and skills (Berry and Sahlberg[32]).

Although the emphasis is on group work and interaction, the teacher still has a meaningful role in planning the learning environment and explaining the essential concepts and basic principles. The role of a teacher in co-operational work is to create a learning community. A teacher creates the conditions within which the students can construct the meaning from the material studied, by processing it through the existing cognitive structures and then retaining it in their long-term memory, where it remains open for further processing and possible reconstruction. Learning is conceived of as something that a learner does, not something that is done for a learner. Students activate their existing cognitive structures or construct new ones to subsume the new input. Through teacher actions the students' competencies and talents are developed. Education is also a personal transaction among and between the students, as well as between the teachers and students as they work together (Johnsson and Johnsson[33]).

We learn mathematics by doing and using mathematics. Speaking and discussing are important elements of mathematics. The skills of problem-solving and learning the concepts can be strengthened by discussions between teacher and pupil, as well as between the pupils (Berry and Sahlberg[32]).

Playing learning games

In this article, the terms 'learning game' and 'teaching game' are considered synonymous. A learning game is defined as a game in which the focus is on practising concepts, numbers and facts and the relationships between them. Playing learning games is a natural way of working for a child, and one way to make the mechanical counting practice interesting. It is also evident that through playing games both problem-solving and deduction skills can be developed. In itself, playing learning games is motivating and brings excitement to the mathematics lessons. A learning game also provides challenges to the more skilled as well as to lower-achieving learners (Pehkonen and Pehkonen[34]).

There are several reasons for using learning games in mathematics teaching. General facets of learning such as concentration and observation, considered and impulsive action, flexibility and adjustment, logical and systematic thinking, creative thinking and different forms of communication can be promoted. The study results emphasise the importance of functioning: 'I do and I understand.' Playing the game gives pupils more space: they can study in their own rhythm. The teacher does not hurry them or disturb them with questions. Learning games also provide immediate feedback to pupils. With learning games, the contents can be taught and rehearsed, a positive atmosphere can be created and social interaction can be promoted in a classroom (Pehkonen and Pehkonen[34]).

2 MEP: early innovations

Yvonne Beston, *London Southbank University*

Our school context and background

In 1999, Hillside Primary School in Orpington, Kent was founded through the amalgamation of an infant and a junior school. From the start we were eager to establish a clear focus on basic skills. I took the role of headteacher late in my career after years of teaching experience. My previous experience had provided me with a sound basis for decision-making along with the confidence to investigate new methods and a belief in taking the school forward in trialling new methods. My personal philosophy as a teacher and headteacher is that it is important to research, enquire and read in order to find new successful approaches to teaching and learning. Consequently, as a school leader I encouraged my staff to take part in courses across the country and I also attended national conferences and seminars. Engaging in independent research and professional development opportunities provided us with autonomy with regard to curriculum development rather than a reliance on the local authority.

Introducing MEP

Independent research into results of international mathematics testing and league tables demonstrated that Hungary had strong results in such tests. Based on these findings, I decided to develop links with Professor Burghes who had developed a research project, the Mathematics Enhancement Programme (MEP), which was based on Hungarian-style teaching methods. Aspects of MEP were introduced at staff meetings by Professor Burghes in 2003 and included direct teaching, pupil discussion, explanations and demonstration, use of resources and the suggested lesson plans. Our staff were also provided with examples of good practice, and videos were used to demonstrate the ideas. These included lessons ranging from Kindergarten to Year 5: the high level of mathematical skill, understanding and sheer enjoyment shown by the Hungarian children in all year groups was immediately apparent.

The initial reaction of staff was enthusiastic; however, concerns were raised as to how this way of working could be translated into our current context. Our results in mathematics had been low since the amalgamation, with only small rather than substantial rises. Consequently within the context of the English schooling system this meant that we were required to work closely with local authority advisers. Within my leadership role, I had the confidence and the enthusiasm to initiate change; however, it was necessary also to motivate the staff. The older teaching staff were somewhat resistant to change and for younger staff change could be a risk in terms of their career progression as they were unsure how both the local authority and then future employers would view this innovation. Undertaking staff development in such a way was both new and challenging as generally schools would seek support from the local authority first, sometimes exclusively.

Visiting Hungary

To further promote understanding and experience of this way of teaching, a visit to Hungary was arranged through funding from the research project during the May half-term holiday in 2003. The group included myself and the two assistant headteachers, one of whom was the mathematics co-ordinator. This visit enabled us to observe a range of classes and see the pride of the staff in their high-quality teaching. There was evidence of direct class teaching, and the key aspects that we observed were:

- Pupils were encouraged to explain their thinking to the class.
- Resources were available for each child.
- Concepts were developed, reinforced and made relevant.

The lessons were conducted at a vigorous pace, which resulted in less down-time than occurs in English schools when the main activity is under way. There were also key differences in the structure of the lesson planning. Rather than the three-part lesson – comprising a starter, main activity and plenary, which had become central in England – lessons consisted of a number of varying activities with mini-breaks. This enabled skills and concepts to be readily revisited and reinforced. Another difference was the direct whole-class teaching rather than group teaching. This allowed all pupils in the class to move forward as a group and I saw the benefits in this – rather than labelling pupils and not providing them with opportunities to visit and revisit certain topics. Differentiation and catering for special needs pupils was an area of interest and concern for my staff. However, in Hungary we did observe a number of special needs pupils taking an active part and learning during the lessons. We were also advised of a number of special schools in Hungary which cater for children with diverse needs. Additional reinforcement opportunities are also available in the afternoon in Hungary as the school day is arranged for the basic lessons to cease at 1pm. The afternoon is then available for wider curricula such as music and art and also reinforcement opportunities for those who require it.

Implementing MEP

On our return from Hungary, the senior staff were motivated to implement MEP, although a number of reservations were still apparent. The foremost concern was the proposed classroom layout in Key Stage 1 with desks placed in rows to enable whole-class teaching. This change required a lot of encouragement as the Key Stage 1 curriculum within England has been strongly dominated by a group-work ethos for many years. However, it was my perspective that we should look beyond the prevailing culture and see what could be possible; therefore I suggested that furniture could be moved for these sessions and then returned to its former position for the rest of the day.

The detailed plans, which provided a breakdown of each activity and process, were an aspect of MEP which appealed to me as a school leader. However, it also required a commitment to implementing the MEP scheme, as there was a need for a significant investment of time to investigate the lesson plans provided by the scheme.

Overall, in leading the school towards change, I needed a scheme that I believed in and which a) provided support for newly-qualified teachers so that they could learn to teach from a sound platform, and b) enabled concepts to be clearly developed, with reinforcement provided. The main body of the staff needed a scheme providing continuity so that from year to year we could build on concepts, revisit skills, ensure progress and promote the enjoyment of mathematics for both pupils and staff.

An initial choice for our school was whether to gradually introduce MEP from Year 1 or to launch it throughout the whole school. To ensure that all staff and pupils were involved from the beginning, we decided to launch it throughout the school. We were aware that more progress could be gained for the younger year groups who would work with MEP for a longer period. As MEP was introduced throughout the school we were also working with two local authority advisers who were focused on improving the targets within the Year 6 cohort, although our school staff did not expect immediate results. An important element in introducing the new mathematics programme was involving both the parents and governors of the school. We organised presentations for the parents by Professor Burghes and this generated both enthusiasm and support. Additionally support was received from the school governors and they also attended the parent workshops, staff meetings and INSET provision.

A positive feature of implementing MEP was the opportunity to receive the resources at no cost to the school. Rather than textbooks, these consisted of lesson plans for Years 1 to 6 and associated practical aids such as numberlines, posters and number cards. Pupil workbooks were also provided and used to assess pupils' progress at the end of each lesson. Additionally pupils were involved in marking their work throughout the lesson, which ensured that the pace of the lesson was maintained and provided a quick overview of the progress of the whole class.

As we implemented MEP across the school, a number of issues began to emerge. An initial concern was the need to use lesson plans that were targeted for pupils one or two years below our year groups. Although our standard assessment test (SAT) levels were improving, our pupils' levels were still much lower than those in Hungary. To address this, we encouraged staff to plan at a level which was appropriate for their year group. In my leadership role, I supported staff to use the materials suitable for their pupils' achievement and observed lessons regularly to ensure that we were working at appropriate levels for our children. For example, in one lower-achieving Year 4 set the children were working on Year 2 MEP materials.

Different levels of support were available to support the process of change. As a school leader I was provided with support from constant contact with Professor Burghes and his colleagues. We were also able to contact other schools involved in the project and visit them to compare experiences. Support for staff across the school also came from visits by Professor Burghes and colleagues to undertake supportive observations and lead staff meetings which provided opportunities for staff to share concerns. These observations did highlight the need for additional support for some staff and this was provided by myself and my mathematics co-ordinator. Further opportunities were also provided for the wider teaching staff to visit Hungary and observe the teaching from which the scheme was developed. This generated enthusiasm in those staff who were able to take the opportunity.

Valuable links were made with a local grammar school and a primary network was formed along with four other local primary schools. The grammar school had acquired specialist status for mathematics and IT, and our leadership team attended meetings and shared our experience. We used their ICT suite, enjoyed parent workshops led by their staff as well as a mathematics day organised for all the primary Year 5 pupils. These links resulted in the grammar school beginning to use the secondary MEP materials. Furthermore, other local primary schools also showed a keen interest in the MEP project. Funding was provided for supply cover so that staff from these schools were able to observe our lessons and to speak with our staff. This activity resulted in some more local schools joining the MEP project.

Professional development through lesson study

As our experience in implementing MEP developed, we were encouraged to widen our view of international work beyond Hungary. Further work involved investigating Japanese teaching and professional development. At this time, opportunities were provided for a number of UK schools involved in the project to send their mathematics co-ordinators to Japan to observe teaching and, more significantly, lesson study. Lesson study is a professional development process whereby teachers work together in year groups or key stage groups to plan, teach and observe lessons critically and constructively. Our mathematics co-ordinator joined the group to visit Japan and was able to share the experiences with colleagues. At this point, we shifted from simply implementing MEP maths to investigating the process of lesson study. Within the UK, teachers are often inhibited by observation and therefore it was important to convince staff that this was to be a positive experience. We began by organising groups of teachers to look at the lesson plans in detail: after this the lesson would be taught by one teacher while the others observed and then a debrief would take place where we would discuss how the lesson had gone. This proved to be a rewarding experience for our staff who were enthusiastic to share their ideas with others.

This form of school-based collaborative practice has proved valuable in changing hearts and minds, and has a key role to play in enhancing primary mathematics, where you are wanting teachers to try out and evaluate new approaches (and this is illustrated in Section 5.2).

3 Implementing MEP: lessons to be learned

Melanie Hazell, *Long Lane Primary School, Reading*

Our school context

Long Lane is a two-form entry school with approximately 280 pupils of a diverse range. We have a Foundation class following the Early Years Strategy, a one-form entry Year 1 class and a Year 2 class, with 30 mixed-ability pupils in each class. From Year 3 to Year 6 the children are taught their daily maths lesson in year-group ability groups, a lower and higher set for each year group.

Long Lane historically has a record of good mathematics scores in national testing both at Key Stage 1 (KS1) and Key Stage 2 (KS2). Before we began using the Mathematics Enhancement Programme (MEP) resources, mathematics was deemed to be going adequately well and particularly well for the higher achievers. However, examination of the children's progress through use of national and school-based data highlighted that the children who were not making enough progress were the lower-achieving children. In response, various schemes and intervention programmes were put in place but none of these appeared to make any significant difference to those children. Our headteacher wanted a radical shift in mathematics teaching and sought to put in place a programme which would continue to challenge and motivate the higher achievers, raising our percentage of children gaining a Level 5 whilst also boosting the lower-achieving children and raising the percentage of children who were able to gain a Level 4 at the end of Year 6 (age 11).

Introducing and implementing MEP at Long Lane: the first year

Both independent research and consultation with the senior management team were undertaken, and it was decided to trial a model of mathematics teaching based on the European style such as that used with MEP. This model aimed to challenge all abilities and, by continually repeating concepts over and over again, the expectation was that the lower-achieving pupils would make better progress and attain higher scores. Following the decision to use MEP, all the teaching staff were given an introduction to the model by the headteacher and by Professor Burghes. We were able to look at the practice books and see all the online materials that supported the programme. We were all encouraged to explore and experiment with the materials and see how we got on.

Some of the initial concerns about using MEP included how the practice books should be allocated to particular year groups. We questioned whether the lower set in Year 3 should start on the Year 2 book and the higher set in Year 3 begin on the Year 3 book. Another area of concern with the older children was that the Year 6 children would be beginning a new maths programme when they had not experienced the teaching and learning within the programme previously. Of equal concern for the Year 6 teachers was the pupils' performance in the national testing. It was questioned how long pupils should be working on a new programme rather than revising and preparing for the upcoming SAT exams.

There were also positive aspects to the initial introduction of MEP. It was viewed by several members of staff to be a new and challenging programme that would support our children and raise their attainment and progress. One of the most positive aspects highlighted by some of the teachers was the supporting material that MEP provided, which included detailed lesson plans and interactive whiteboard materials. For many of us, we felt that the pressure was off us having to find challenging activities as MEP had already laid out the activities for us. However, on reflection I think that this may have been an initial misconception and a reason why for some teachers the introduction of MEP did not run as successfully as it could have done. The focus on maintaining a good pace in lessons whilst using every activity in the books and lesson plans resulted in teachers moving from activity to activity regardless of the children's understanding or completion of the work.

As MEP was implemented in the school some teachers flourished and really enjoyed teaching the programme. These teachers developed their practice and learned through trial and error that the lesson plans did not need to be followed as a script but, as with any scheme, there was a need for teachers to insert their own style and interpretation of the materials to cater for the learners' needs. In the junior area of the school, puppets were used to help the children interact with their teacher and explain their mathematical thinking. Further up the school, in the classrooms where use of the material from the programme was working well, there was more discussion of mathematical ideas and reasoning, a greater use of mathematical vocabulary and higher levels of enjoyment for the children. However, in contrast, some teachers were finding it very difficult to use the materials and a strict focus on covering every aspect of the activities meant that only a small part of the lesson plan was covered and there was a laborious focus on smaller issues. This was largely a concern for the teachers working with the lower-achieving children, as they felt the proposed pace was too quick and the concepts presented to the children were at too high a level for them to understand.

In retrospect, at this point in the implementation process, there was not enough done to listen to the concerns of these teachers and to help them find ways of moving forward through adapting the material to suit their group of children. However, I also believe that it was a difficult and challenging task for some of the teachers to try and change the fundamental way that they teach. Long Lane has a number of teaching staff who have been at the school for many years and have always taught in the same way. In contrast, some teaching staff were arguably more ready to accept change and challenge and try out new ideas. The introduction of MEP might have been more successful through engaging in whole-school professional development and developing forums in which concerns could be shared and confronted as a staff. However, following the initial implementation of MEP, our teaching staff became divided between those who found MEP challenging and engaging and those who disliked the content and the way they perceived that it had to be delivered.

Changes to implementing MEP: the second and third year

After using the MEP material throughout the school for two and a half years, the first major shift came from the Year 1 and Year 2 teachers. They felt that the children had become despondent with material as it was presented in the books. In their view, the pages were dull-looking and didn't grab the children's attention enough. Whilst the teacher-led activities and active involvement of the children were working well, the teachers' perception was that the children disliked completing the independent activities in the books. A decision to stop using MEP and begin using Abacus Evolve was made by the Key Stage 1 team. They chose to use these resources as they felt that the books in this series were more engaging, colourful and inviting for the children to use. They did, however, want to maintain the pupil voice element of the lessons that MEP promoted, whereby the children were more active in discussion, explaining their thinking on the interactive whiteboard.

A further complication within the context of our school was that at KS2, our local infant school feeds into our classes. These children had not been in classrooms where the MEP programme was taught and therefore were entering our KS2 classes with different experiences. The mix of children who had been within classrooms where MEP was used for two years and were familiar with the pace and structure alongside the children who had not had this experience made the teaching of the Year 3 groups problematic. Consequently, the decision for KS1 to use Abacus Evolve was cemented. MEP material would be used from KS2 within our school so all children would begin the programme together.

As we continued to use the MEP material, the teachers who were comfortable and confident with using and adapting the material successfully maintained its use in their classrooms. However, a number of teachers continued to hold strong reservations. The main concern was the continued poor performance of those children who we had been hoping to target. While the average and high-achieving children were making progress in mathematics, the lower-achieving children continued to fall further behind and failed to make sufficient – or in some cases any – progress. School-wide discussions to address these concerns were undertaken and a first step was for some of the teachers who taught the lower-achieving sets to observe and discuss practice in the higher-achieving sets to see if there were any aspects within the lessons which could be used to support the children they taught. In-depth discussions also centred upon the level of material within the books and whether the children should be taught using books from earlier year levels. There was agreement that teachers needed to alter and adapt the material to suit the needs of the children. However, despite this, for those staff who were negatively inclined towards the continuing use of MEP there was little change.

The lack of progress made by the lower-achieving children within our school was of great concern to both the senior management and teaching staff. It was therefore felt that decisions needed to be made to confront this problem. At this point under the leadership of a new headteacher and some input from our local authority, an inspection of mathematics teaching was undertaken throughout the whole school. The findings were mixed with lessons ranging from 'inadequate' to 'outstanding' or 'with outstanding elements'. It was apparent that even with the introduction of the lesson plans and MEP material, we were in the same position that we had previously been in, with some parts working really well whilst others were not. At the time of the inspection all KS2 classes were using MEP; however, it was noted that there was little consistency from one class to another. For example, some teachers were following MEP as a strict script, page by page and activity by activity: in contrast, in other classrooms the material was being adapted and developed. There were also a number of differences in teaching practice with more investigation and exploration of mathematical ideas in some classes and a great deal of variation in terms of pupil talk and interaction. Another difference noted was the recording of evidence for assessment and notes on what aspects of each lesson had worked well or needed further work.

Use of the information gathered from the inspection process formed the basis for discussion of further changes. A large number of questions and suggestions concerned altering the grouping arrangements. For example, the following questions were raised:

- Should we continue to set the children by attainment?
- Would the implementation of MEP have been easier if we had begun by teaching the children in mixed attainment groupings?
- Would we have succeeded in improving the attainment of the lower-attaining children if we had considered altering the grouping arrangements?

As an alternative to changing the grouping arrangements, it was decided that the teachers who had successfully implemented and adapted the MEP material would take the lower-attaining sets. For example, I moved from teaching the upper Year 3 set to the lower Year 5 set. This class included children who were deemed most at risk and in terms of assessment results, some of the children had made little or no progress for two years. This was a challenging experience as the cycle of low attainment needed to be addressed with a move towards pupils progressing successfully. To achieve this I needed to reflect on my position as a teacher, as mathematics co-ordinator and to analyse how to successfully facilitate the pupils to progress.

Reflections on my personal journey in implementing change in my mathematics teaching

Paramount in my development has been: engaging in research within my own classroom to facilitate children's learning; providing opportunities for children to explore their own thinking and ideas; facilitating pupils to talk to their peers about their mathematical thinking; and developing a view of mathematics not as a black and white subject but as a technicolour rainbow. The provision of engaging tasks and activities lays the foundation for children to aspire to raise their own levels of achievement and succeed in mathematics. MEP provided a structure along with activities and lesson plans. However, my development and interpretation of the materials and knowledge of the learners I taught was equally important. While the model for teaching may be provided through schemes such as MEP, it is still the classroom teacher who has to make the difference to children's learning.

When we work with children with special educational needs, it is important that we strive to understand the gaps in their learning. While some concepts may cause particular difficulties, other concepts may be easily developed. Therefore, it is important to encourage children to value the aspects of their learning in which they are achieving – this will provide motivation for them to overcome barriers in more difficult areas. With all children we must celebrate individual success; fostering a positive and safe classroom environment supports children to raise their own expectations. For those children who come to our lessons with emotional and behavioural difficulties, simply knowing that their teacher wants them to achieve and succeed can be a motivating factor for progression in mathematics. By providing boundaries for behaviour, selecting children to model appropriate peer discussion and encouraging groups of children to work collectively, it is possible to develop a cohesive, non-threatening environment in which all pupils may progress. Importantly, children need to develop an understanding of mistakes as a way of learning and moving their understanding forward. Rather than asking pupils to re-do a page which has been marked and is full of errors, carefully scaffolding discussion and facilitating input from the varying points of pupil understanding in the classroom can help to correct misconceptions.

Involvement in two research projects, the first focused on developing early algebraic reasoning and the second investigating the use of lesson study, has supported me to develop the recognition that mathematics lessons do not need to always follow a rigid structure involving formal groupings based on attainment. In contrast, it is effective to use a range of grouping structures such as pairing children according to the learning objective, grouping children together who are at different attainment levels to scaffold each other's learning or asking children who are struggling on a particular concept to work together. My own teaching practice has developed and evolved through reflecting upon and varying my personal practice. I believe the freedom to reflect and change my own practice has supported me to find ways of engaging and inspiring my pupils which then result in raising their performance.

A key area of learning in my own development through involvement in the research projects has been that within the classroom, the teacher's voice should not always be the dominant one. Within the classroom it is paramount to enhance pupils' mathematical thinking and learning through fostering an ethos of effective talk. This can be achieved through facilitating talk partnerships, small-group work and a specific focus on questioning techniques. It is no longer sufficient (and arguably never has been) to rely on a traditional model of the teacher explaining a concept and the children rehearsing, to adequately develop mathematical thinking and learning. Children need to be provided with opportunities to explore their own and one another's thinking. Developing the strategies to explain and justify your own thinking both supports other learners in the classroom but also helps the pupil who is doing the explaining to develop their own conceptual understanding and reasoning.

In a personal sense, during my schooling experiences I both struggled with and feared mathematics. Fractions were an area that I never really understood and it was not until I had to revisit such concepts when training to become a teacher that my eyes were opened to the connections between fractions and decimals and their relevance in real-life situations. Teaching the concepts supported me to truly understand them and gave me an understanding of how important it is to break ideas down, understand pupil misconceptions, provide time for pupils to make sense of concepts and work through their ideas, and facilitate understanding through the use of careful questioning. This has parallels to teaching within mixed attainment groupings where children of different attainment levels work closely together. It could be argued that the child of a higher attainment level would be constantly helping and teaching their lower-attaining partner, however, working in a mixed attainment setting provides children with opportunities to explain their thinking, break strategies down into smaller steps and therefore develop a deeper understanding of concepts.

Within my own classroom, I am aware that MEP provides ample opportunity to develop such classroom practice and also has an excellent array of opportunities to explore early algebra. Furthermore, it is excellent at introducing and revising key mathematical vocabulary and mathematical talk within lessons which facilitates pupil attainment. However, using the opportunities that MEP offers requires learning both on the part of the teacher and pupils. Creating a classroom environment which focuses on children's mathematical thinking and effective talk requires careful modelling by the teacher along with specifically highlighting pupils who are modelling the appropriate behaviour. Children have to be encouraged to learn to listen to each other carefully and then reflect on the idea when a peer has finished their explanation. Crucially they must learn to respect ideas and viewpoints which are different from their own. Providing children with the time to discuss their ideas has led to my pupils beginning to question both their own thinking and the thinking of others. This supports them to embed their developing knowledge and enhances their progress. There is also a need to provide the children with rich opportunities to explore and question in a variety of situations; along with giving them the autonomy to choose suitable equipment to demonstrate their ideas and reflect on their own thinking, with questions such as 'Is this always true?', 'Is there a pattern?', 'What happens if …?' In such classrooms, the teacher's role shifts to that of a facilitator who visits groups and individuals while they are engaged in discussions, listens to their ideas, assesses their reasoning or misconceptions and then may prompt their thinking through questioning. Through facilitating whole-class discussions where children are chosen to share their ideas in a structured way, learning takes place for children at varying attainment levels. Embedding such a model also supports teaching to address misconceptions. The teacher is able to plan the lesson in a way that allows misconceptions to be explored and resolved through careful questioning and listening to individual responses.

Reflecting on my experiences in implementing MEP to try and raise pupil achievement highlights a number of key areas of importance in enhancing mathematical learning. Quality materials and tasks alongside an effective classroom environment are essential to raise progress. However, there is also a need for both motivation and a positive attitude towards mathematics from the teacher. Another key element in my personal success in implementing MEP within my classroom has been involvement in the professional development offered by the research project on lesson study. For me, this was a significant opportunity to learn collectively within the team in which I worked by planning, delivering and analysing lessons together. This was a shared rather than individual process and was far less intrusive than a formal observation to grade me as an individual. In contrast, during the lesson study work we were able to focus on pupil interaction, content and resources, and collectively analyse how the lesson could be improved. This provided an excellent opportunity for professional development and was a successful way to observe pupil interaction and their progress across both cohorts and phases.

Ultimately, as educators, we want the children we teach to be the best that they can be. It is important to equip children with a sound knowledge base in all areas of mathematics. As primary teachers our job is to seek the best way to inspire, guide, motivate and celebrate children's progress in mathematics so that they move into Key Stage 3 as confident, thriving, keen mathematicians. In my view, schools that choose to use MEP have a valuable resource bank to draw upon. However, it is important to develop models of school-wide professional development to support its implementation and also to work as a team to address any concerns or difficulties with the material. While our school has decided for a number of reasons not to continue using it, in a personal sense I have learned valuable lessons from my experience with MEP and many parts of the model are apparent in my mathematics teaching.

4 Moving forward with lesson study

Derek Robinson, *Head of Mathematics, Bishop Luffa School, Chichester*

What makes a great lesson?

Perhaps surprisingly, at least at my own school, there is general agreement between both teachers and pupils as to the answer to this question. We all agree that great lessons should be interesting, thought-provoking, enjoyable and, wherever possible, contain a 'wow factor' that both pupils and teachers take with them as they leave the lesson. It is even better if the enthusiasm generated can be taken home and shared with parents and carers. Nothing will raise the profile of mathematics more quickly than children going home full of excitement, telling their parents how much they enjoyed maths today and sharing what they have learned.

Therefore, it was really disappointing to hear that within England, despite our proven gains in achievement in mathematics at primary level, our pupils' enjoyment of maths had plummeted (Sturman et al.[35]). The TIMSS results showing higher attainment yet lower enjoyment may be related to the 'daily maths lesson', a key feature of the National Numeracy Strategy (NNS). This has been well received by teachers, yet research studies, for example, by Kyriacou & Goulding,[36] have found that it has failed to generate the higher-quality dialogue, discussion and strategic thinking intended. These findings raise the question of how we may maintain high attainment and, at the same time, impart a lifelong passion for mathematics in our children. Evidence from other countries suggests that sustainable change is not achieved through externally driven reforms but generated from within our own schools and delivered by individual teachers in their own classrooms. In other words, once the classroom door shuts, it is the teacher who has to generate the love for maths that is missing in many of our primary classrooms.

School improvement relies on *learning to do the right things* in the setting where you work rather than what you know when you start to do the work (Elmore[37]). In other words, as Guskey[38] surmised, effective professional development should be an ongoing and an embedded process within your job. There is evidence of the common factors necessary to improve student performance from the findings of Barber and Mourshed[39] as published in the McKinsey report *How the world's best-performing school systems come out on top*. In this report Barber and Mourshed examined 25 school systems worldwide, including the top ten performing systems. The authors concluded that successful schools need to do three things:

- get the right people to be teachers
- get the best out of these teachers
- step in when pupils start to lag behind.

Their findings indicated links between the quality of school systems and the quality of teachers and a recognition that outcome was improved through improving instruction. Top-performing countries have systems in place that ensure that when a teacher enters the classroom he or she has the knowledge, the capability and the ambition to deliver the best possible lesson to ensure all children learn. To achieve this, a clear definition of quality instruction needs to be established alongside professional development which provides teachers with the capacity and the knowledge to consistently deliver great instruction.

So how do we ensure that when a teacher enters the classroom he or she has the knowledge, the capability and the ambition to deliver the best possible lesson that ensures all children learn? And perhaps, even more importantly, whose responsibility is it to make sure this happens?

Continuing professional development and school improvement

Elmore[37] argues that the responsibility for change lies within each school. However, while it is important for teachers to have a stake in their own learning, it does not follow that they will become better teachers or that they will be able to recognise and replicate great instruction on a daily basis.

Developing models of CPD where teachers learn from each other, learn from what works and learn from good practice in other countries offers one possible approach to long-term school improvement. Often in the UK and the United States, teachers work alone and teaching is an isolated profession which takes place behind closed doors (Barber & Mourshead[39]; Stigler & Hiebert[16]). However in high-performing countries such as Finland and Japan, a collaborative approach is taken whereupon teachers jointly plan lessons, observe and support each other to improve (Stigler & Hiebert[16]). This is exemplified by the model of professional development in Japan entitled 'lesson study'.

Within England and the USA, there has been interest in lesson study as a possible model for improving instruction (Dudley[40]; Lewis & Perry[41]; Yoshida[42]). However, while many lesson study groups have been formed, research evidence on their effectiveness is still limited. One difficulty in developing models which are derived from other cultures such as lesson study is the process of teaching as a cultural activity with teachers following cultural scripts (Stigler & Hiebert[16]). Prospective teachers are influenced by their own schooling experiences and consequently teach in a similar way to the way they themselves were taught (Bartley[43]). Therefore it is important to learn how lesson study can be used as a vehicle for sharing good practice and essential that a clear definition of effective teaching practice is developed and shared. Evidence from research into highly effective countries in mathematics education such as Japan indicates that in these countries problem-solving is seen as an essential part of the mathematics curriculum (Stigler & Hiebert[16]).

For further information on Japanese lesson study, reference can be made to the publication *Lesson Study: Enhancing Mathematics Teaching and Learning* (Burghes & Robinson[17]). However, for added clarity it is appropriate to reiterate some of the key ideas from this publication. See section 2.1 (pages 23-24 above) for further examination of Japanese lesson study.

Japanese lesson study

In Japan, lesson study is the major form of professional development for all primary and many secondary teachers. Its goal is the continual improvement of teaching within a school so that their own pupils will learn more. One of its main aims is to improve the teachers' ability to understand how their own pupils think and learn. Lesson study is situated within the model of 'learning from practice' and an underpinning belief is that teachers can learn and improve their own practice from observing others teach (Isoda[44]; Lewis[20]). It involves small groups of teachers coming together to plan, observe, reflect and refine lessons for a unit of work (Lewis[20]). It assumes that teachers learn best through working collaboratively with their colleagues in their own environment (Lieberman[45]).

Effective CPD improves classroom practice through enhancing the professional skills and attitudes of teachers so that they improve student learning (Joyce & Showers[46]). The basic philosophy of lesson study is that evidence of effective classroom practice can only be found in the classroom. Lesson study is best described as an ongoing journey that is directed by its interaction with its setting which includes the long-term goals of the school, the teachers and the students. The process begins as a form of backward design by establishing the goals and purpose of professional development, and for lesson study the final goal is what we wish our pupils to be when they leave our school (Guskey[38]; Wiggins & McTighe[47]). This is described as an 'overarching aim' and within the lesson study process it is important that in the initial stages a research theme or overarching goal is established to guide the process. This should be child-centred and should relate to the school's vision of what kind of students they want to produce in their school (Tanaka[48]; Yoshida[42]). Establishing a lesson study goal is the first thing teachers in Japan usually do and, by preference, it should be a whole-school vision and be kept for several years. The lesson study process is about bringing this goal to life in the classroom.

Most schools in the UK have snappy mission statements that adorn school websites and noticeboards but are rarely used to drive learning. In contrast, lesson study can act as a vehicle to bring a mission statement to life (Lewis & Perry[41]).

For example, at Bishop Luffa School the overarching aim of the mathematics department is:

'Our students will become independent thinkers (learners) who enjoy working together to produce creative solutions in unfamiliar situations'

As you can see, this is not a specific mathematics goal and variations are now used in other subject areas across the school. In mathematics, our planning is guided by the changes we wish to bring about in our pupils. Our challenge then becomes to design series of lessons that will enable our pupils to:

1. ***Enjoy doing mathematics*** *– to help students learn to enjoy and sense personal reward in the process of thinking, searching for patterns and solving problems*
2. ***Gain confidence and belief in abilities*** *– to develop students' confidence in their ability to do mathematics and to confront unfamiliar tasks*
3. ***Be willing to take risks and to persevere*** *– to improve students' willingness to attempt unfamiliar problems and to develop perseverance in solving problems without being discouraged by initial setbacks*
4. ***Interact with others to develop new ideas*** *– to encourage students to share ideas and results, compare and evaluate strategies, challenge results, determine the validity of answers and negotiate ideas on which they all can agree.*

The effectiveness of our lessons is then measured against these four objectives as well as the mathematical goals of the lesson.

When our department initially began using lesson study, we had the common belief that its purpose was to design perfect lessons; however, we rapidly realised that lesson study is better described as an ongoing co-operative research project into how children learn mathematics. Rather than focusing on producing the 'perfect lesson', instead the process involved the teachers in each group working together to plan, test and then reflect on lesson sequences which were specifically designed with the aim of improving learning (Tall[49]). It should be noted, however, that during the lesson study process, attention is focused on one particular lesson in the sequence, called the research lesson, which is chosen carefully as one of the key lessons in the sequence.

Lesson study provides a possible model to deliver effective professional development, but what is not immediately apparent is how this can also help with our understanding of effective instruction. To understand this more fully, it is necessary to look more closely at how Japanese educators use 'structured problem solving' to drive learning.

Structured problem solving

The problem-solving orientated lesson is a common approach used in Japanese mathematics lessons. These lessons involve collaborative work by students which is carefully guided by the teacher (Kishimoto & Tsubota[50]). Japanese problem-orientated lessons often follow a similar structure where both the teacher and the students have well understood roles (Natsusaka[51]; Tanaka[52]; Yamamoto[53]). Isoda[44] describes these roles as follows:

Phase 1: Presentation of the problem *(about 10 minutes)*

- The teacher presents the problem in a way that the students can easily understand and know what is expected of them.
- The students work out what the problem is about by reading it themselves, listening to the teacher's instructions and discussing it amongst themselves. They check similarities and differences between what they already know and what they need to learn today and begin to develop a perspective on ways of tackling the problem.

Phase 2: Developing a solution *(about 15 minutes)*

- The students think about the problem on their own and try to find solutions on their own.
- The teacher purposefully walks around looking at the students' work, making notes and deciding in which order to ask the students to present their ideas in phase 3. The teacher in general does not guide the students apart from giving hints to students who cannot make progress on their own. In this way the maths being created belongs to the students.

Phase 3: Progress through discussion *(about 10 minutes)*

- The teacher asks three to five students who used different methods (or got different answers depending on the type of openness being used) to explain their approaches to the rest of the class. The teacher remains neutral to the ideas.
- The students listen to the explanations and try to reach a common understanding of better solutions by discussing the strong and weak points of each approach proposed and identifying what they have in common.

Phase 4: Summarising *(about 10 minutes)*

- The teacher summarises the group findings and in particular emphasises the important points addressed in the lesson. The teacher will challenge the students with similar or developmental problems for homework.
- The students often write down what they have learned in their journals.

The problem-solving approach used in Japan has been described as structured problem-solving and the overall process is entitled the 'open approach'. Isoda[44] describes the open approach as having three aspects: 'processes are open (various ways of solving the problem)'; 'ends are open (various answers against an open-ended problem)' and 'problems are open (changing and developing problems from a given problem)'. Each of these three approaches is underpinned by the belief that students solve problems not only to apply mathematics that they have already been taught but to learn new mathematics.

There are some key aspects of teaching through the problem-solving approach used in Japan. Firstly, teachers share both a common understanding of the approach used and also use specific pedagogical terms to describe and discuss their roles in the process. The problems to be used in the lesson are specifically and purposefully selected to drive rather than test learning. According to Stigler and Hiebert[16] this is common across high-achieving countries. In such an approach, the choice of the problem is critical and its design needs to ensure that all students will have some success in solving it and flexible approaches can be used (Becker & Epstein[54]; Fernandez & Yoshida[55]). A key aspect for teachers in planning the lesson is anticipating student responses and to support this, the teachers write down all anticipated student responses – which also supports their growing capacity to see lessons from the students' perspective (Becker & Epstein[54]; Lewis[56]). Additionally, problems are presented to the students to be solved rather than the teacher demonstrating how to solve the problem first (Stigler & Hiebert[16]). Consequently, learning begins with a problem to be solved, and the problem is posed in such a way that students need to gain new knowledge before they can solve the problem. Rather than seeking a single correct answer, students interpret the problem, gather needed information, identify possible solutions, evaluate options, and present conclusions. Students are encouraged to take an active role in constructing their own mathematics by communicating with one another and to develop a belief in their own ability to learn and to think (Kishimoto & Tsubota[50]).

To illustrate the ideas above, a group could consider the following questions (Kwon, Park & Park[57]):

A Different Number
Among the following numbers, choose a number which is different from the others.

If possible, try to find many possible cases or answers.

1 2 4 6 8 12

Before moving on, how would you answer this question?

Can you find a reason for each number to be classified as 'different'?

Can you find multiple reasons for each choice?

Initially, do this on your own. Allow 10 minutes for this activity.

Now compare your ideas with those of other members of your team.

Make a definitive list of your answers.

If appropriate, try this with a Year 5 or Year 6 group.

Compare their answers with yours.

How good were you at predicting their answers?

Now think about the choice of numbers. Suppose this question was put to your lesson study group: would you have argued for a change in any of the numbers?

If so, explain why.

Through the design of a problem that naturally extends the students' learning towards the desired outcome, new concepts are linked to previous learning (Tall[49]). Students are able to use their own ways of thinking to solve problems and can be encouraged to discuss and compare their solutions to investigate which are more efficient (Becker & Epstein[54]; Tall[49]). Through focusing on one specific problem during the lesson rather than a large range of examples, the emphasis is on developing flexibility of ideas rather than perfecting a particular skill (Tall[49]).

Choosing the problem

Sawada[18] suggests that the success of the open approach depends very much on the choice of problem. He describes how Japanese teachers firstly determine if the problem is appropriate by asking three questions:

- Is the problem rich in mathematical content and valuable mathematically?
- Is the mathematical level of the problem appropriate for the students?
- Does the problem include some mathematical features that lead to further mathematical development?

They then develop their lesson plan by:

- Listing the students' expected responses to the problem
- Making the purpose of using the problem clear
- Devising a method of posing the problem so that students can easily understand the meaning of the problem or what is expected of them
- Making the problem as attractive as possible
- Allowing enough time to explore the problem fully.

Types of problems

Shimada[58] also explains that we can classify open approach questions into three types:

Type 1. Finding relations. Students are asked to find some mathematical rules or relations.

Type 2. Classifying. Students are asked to classify according to different characteristics which may lead them to formulate some mathematical concepts.

Type 3. Measuring. Students are asked to assign a numerical measure to certain phenomena.

As this approach is common in all Japanese primary and secondary schools, students are familiar with both the wording and style of this type of question.

Moving forward with the open approach and lesson study

Developing the 'open approach' provides pupils with opportunities to be regularly involved in high quality dialogue, discussion and strategic thinking. As teachers develop their understanding of the underpinning ideas of an open approach and develop their own problems they also become involved in high quality dialogue, discussion and strategic thinking with their colleagues. This enables teachers to develop professionally within the setting in which they work (Elmore[37]).

Throughout this paper, there have been three underlying principles, namely that in Japan students are successful in mathematics because schools:

- have a long history of organising their own CPD around clearly identified goals for long-term student change
- have an accepted method of teaching mathematics through problem-solving
- have a means of sharing good practice.

This results in a system which identifies effective instruction and a process to generate, accumulate and share professional knowledge.

David Tall[49] (p.45) argues that 'lesson study has genuine benefits that would be of value to us in the United Kingdom, as long as we think reflectively about what it is we are trying to do in teaching mathematics'.

5 Successful implementation of changes in practice: enhancing the teaching and learning of maths at Grouville School

5.1 The headteacher's perspective

Pam Pitman, *Headteacher, Grouville School, Jersey*

About the school

Grouville School is the largest primary school in Jersey with 358 pupils, plus 31 nursery children. We have a mixed intake of children including a small number of Polish and Portuguese children. We have a transient population as within our catchment area we have bank and hospital accommodation (families come over to the island on five-year contracts).

The context of Jersey

Jersey is a crown dependency with its own government and a permanent population of 91,800. In education we have followed the lead of England, and the Jersey curriculum is a modified version of the National Curriculum.

In recent years, schools in Jersey have been provided with opportunities to undertake varying professional development initiatives. Many schools choose to undertake professional development in mathematics through an initiative entitled 'Making Maths Make Sense' (MMMS). Although our maths co-ordinator attended the initial training, we soon became aware that as a scheme it did not meet the needs of our children. Consequently we adapted elements of MMMS and integrated them with the existing scheme the school was using. However, we were aware that our school-wide mathematics standards were below what they should be; therefore we sought an alternative mathematics professional development opportunity.

Our school context and how it has worked for us

At Grouville School we have developed an ethos of a community of learning which is both sustainable and has the capacity for continual improvement. As a school we established a moral purpose which provides us with clarity, reflects our fundamental principles and enables us to make decisions about what is right for us and what is not. This underpins our decisions when looking at any change or new initiative and rather than looking for 'quick fixes' we actively seek programmes or initiatives that will have positive outcomes in as many areas as possible.

As a staff we have worked hard to develop a community of understanding, honesty and mutual support. We collaborate as a whole staff to make decisions regarding teaching and learning, and the curriculum. This requires that sufficient time is provided for the process of making decisions and there needs to be careful planning in order to create the opportunity for discussion, reflection, planning and reviewing; only then can it be meaningful. It is necessary for all staff to have ownership of decision-making, as this creates a sense of vested interest and ensures real commitment from all staff. Consequently when challenges arise during the process of change, staff themselves strive to seek solutions because in having ownership they want to make it work.

While we were aware that standards in maths needed improving, it was first and foremost important to create the right climate and ethos in our school so that we could ensure full staff commitment. As headteacher I believe that a fundamental part of my role is to recognise and work to people's strengths and talents and empower others to take the lead and make a difference.

Our maths journey

We decided to investigate the possibility of working with CIMT on our mathematics professional development. The first step was for Professor Burghes to visit Jersey for the day and share his thinking and vision with our maths co-ordinator. Over the next two terms Professor Burghes observed maths in action in our school and shared his beliefs of what makes good maths teaching within an international context. Staff watched video footage of maths teaching in places such as Hungary and read research articles about effective maths teaching. Both of these sources led to much discussion with Professor Burghes and between teachers in staff meetings.

An important factor in the effectiveness of our mathematics professional development was a bottom-up rather than top-down approach in which staff were given a package and told to deliver it. It was useful for our staff to develop their understanding of mathematics teaching from around the world and additionally to investigate research findings of effective mathematics teaching. This supported our teaching staff to reflect on their own practice and empowered them to experiment with new ideas. For example, in some classes the teacher experimented with new seating arrangements to promote paired work. The impact of these small changes was apparent very quickly and it was evident that engaging in this professional development led to a school-wide focus on mathematics with many informal conversations happening around the school about mathematics teaching and learning.

As a staff we spent two terms with a focus on good mathematics teaching; trying out new ideas and reflecting on their impact. It was evident that this made a difference both in terms of practice and the children's learning. At this point, a school-wide decision needed to be made as to whether we should continue with the MEP programme or acknowledge that we had benefitted from our journey so far but that we were going to go back to what we had been doing before. By engaging the whole staff in the decision-making process we were able to ensure that all staff took ownership and were prepared to commit to the process of change. After engaging in prolonged discussion, the staff decided that they wanted to implement MEP.

After establishing our school-wide achievement levels in mathematics, we decided to use the programme material from the year below our class levels. For example, our Year 4 classes used the MEP Year 3 materials. The initial implementation of MEP proved challenging and a number of issues arose that needed to be addressed. For example, our Year 1 children lacked the fine motor skills required to write in the small spaces provided in the books; additionally, there was no Nursery curriculum. As such issues arose, solutions needed to be found and changes made to address them.

Both the children and staff found the initial implementation of the MEP lessons demanding. Children were required to think mathematically, reason, use logic and be able to explain and prove how they worked things out. For the teachers, both the quick pace and number of activities in each lesson were challenging. I believe that if you had asked the children at the beginning if they enjoyed MEP, the majority would have said 'no' as it was different, stretching, and hard work! However, we are now in the third year and as the teachers have become familiar with the lesson plans and adapted them as necessary, they have become more confident and creative. Consequently the children are much more engaged and see themselves as being successful learners in mathematics.

Our school policy is to facilitate parental involvement and we ensure that when we are making changes we keep them informed so that they have an understanding of what we are doing and why. This 'open door' policy with its focus on getting parents into school, both formally and informally, has enabled them to see all sorts of learning including mathematics teaching, and ask questions which also lead to interesting discussions.

An important part of the process which has added both breadth and depth to enhancing mathematics teaching and learning has been the collaborative practice model. Use of the collaborative practice model enabled staff to observe mathematics teaching across a range of year groups and develop their understanding of the teaching and learning across the school. Teams of teachers were created from Foundation, Key Stage 1, Lower Key Stage 2 and Upper Key Stage 2. As a group, the teachers collaboratively planned a lesson which was then taught by one teacher while the others observed. Feedback was provided as soon as possible. Involvement in the planning stage meant that the teachers who observed were stakeholders in the lesson. Provision of time and commitment to the value of collaborative practice has led to successful outcomes – which include the enhancement of an atmosphere of understanding and respect across the school. It has been viewed as a valuable process for staff and many have identified it as one of the most useful opportunities they have experienced in terms of professional development. We are now planning to take this model, make appropriate changes and use it as part of our monitoring and evaluation cycle as the principles behind it reflect our beliefs and principles.

Provision of time is an essential element in the process of change. As we focused on enhancing the teaching and learning of mathematics we provided time for:

- trying things out
- discussion and open conversations
- resolving problems and seeking solutions
- celebrating success.

These are all important elements in managing the process of sustainable change.

Both the process of change and our commitment to enhancing the teaching and learning of mathematics have provided us with a framework to undertake professional development in other areas of the curriculum. Our school focus is now on improving standards in English. In the same way as with the professional development we undertook for mathematics, we have engaged in school-wide decision-making to ensure commitment and we have now taken on a scheme that has many aspects similar to MEP with regard to partner work and children taking a peer teaching role. Both these factors empower children to be 'thinkers' and see themselves as successful learners.

Where we are now in mathematics

Enhancing mathematics teaching and learning is an ongoing process. We have been involved with MEP for three years and as it evolves and staff continue to evaluate it critically, new questions emerge along with solutions and change. A recent focus has been the needs of our children, especially in Key Stage 1. Provision of more experiential opportunities before tackling abstract concepts has supported their engagement in mathematics. Another area of focus has been meeting the needs of those children who find it difficult to access MEP. As a school we have investigated the alignment of MEP with National Curriculum levels. Good teaching and learning and our understanding of mathematics thrive in a culture of discussion, questioning and challenge.

Across the school, we have seen our children's achievement levels rise. Our learners make continual progress in the IPMA end-of-year tests and are showing that they can transfer their knowledge and understanding in new situations.

The positive outcomes for the children at our school have developed through staff commitment to the process of change. While using an 'off-the-shelf' mathematics scheme may have been easier, it would not have resulted in the best outcomes for the children. Initially the high quality input from Professor Burghes led to teacher reflection and discussion. Our implementation of the MEP material was supported by an atmosphere of questioning, challenge and change. Finally the process was enhanced by the model of collaborative practice. All of these factors have led to improvements in the teaching of mathematics, developments in staff capacity and strengthening of our community of learning. As a result we have developed classroom environments where children have been empowered to become mathematical thinkers, believing that:

Giving time + staff ownership + commitment and belief + looking for solutions = sustainability

Are we there yet? Of course not… but we have both clarity and sense of purpose about where we are going and what we have to do to get there. That in itself creates more questions and challenges and make us think, further embedding our shared ethos and beliefs.

5.2 The role of the mathematics co-ordinator

Rachel Smith, *Grouville School, Jersey*

Introduction

I have been a maths co-ordinator for about 20 years and during this time I have experienced and implemented many changes. Over the last three years, we have undertaken significant changes in the teaching of maths at our school. This has had a positive impact not only on the teaching and learning of maths but also on the children's use of reasoning, thinking and language in other areas of the curriculum. In addition, there has been an increased sense of collegiality and an improvement in teachers' professional knowledge and understanding.

In this section, I will recount my experiences in managing this change and try and identify why it has worked for us. I will identify both the positive and negative aspects of the experience and some of the solutions to problems that we identified, as well as points to ponder and things to try. The ideas and fundamental principles are not new: this is simply a record of our journey.

Change

As primary teachers we are adept at responding to change. New initiatives come along with amazing regularity only to be subsumed by the next initiative which changes the focus of our energy and absorbs our time. Often as teachers we have been involved in an impressive list of initiatives that have been supposedly full of promise but have amounted to nothing more than a few more files in an over-full cupboard. It is therefore no surprise that some staff are resistant to new approaches and as subject leaders we need to implement any change in such a way that it engages staff in the first instance and then manage that change so that it leads to sustainable development.

In my experience, change that is imposed on staff without an underlying ethos is usually short lived. It begins positively but then hits problems and resistance and does not emerge as sustainable in the long term. There are key points to consider when managing change so that it emerges through this difficult stage towards sustainability.

- **Community** – the ethos of the school needs to be based on a shared understanding. A culture where staff freely share their ideas and feel safe to challenge will allow all staff to work together to overcome difficulties.
- **Shared belief** – staff need to be part of the process from the beginning so that they believe in what they are being asked to implement. Engaging staff from the beginning prevents doubtful staff from becoming defensive of their current practice and the change being derailed. Before implementing any change it is worth considering how individual staff will react and plan for this. Change will be more effective if staff are provided with an element of control: it is about keeping everyone moving forward together.
- **Time** – enough time needs to be given to allow staff to adapt, explore, challenge and assimilate new ideas. The pressure of other subjects can mean that changes are pushed through very quickly. Enough time needs to be given for problems to be explored and for the whole school to develop a consistent approach and a shared vision. This is not a quick fix: it is a gradual process of change.

On a recent course, I was given the metaphor of the slight changes that a pilot makes to his flight path. When you look in the sky, the jet stream looks straight but what the pilot is doing is making a series of gradual changes to his course that when added together lead to a huge shift in direction.

It is this gradual process of change that we have undertaken at our school. The overall shift in ethos, teaching and learning has been significant but the journey has been one of minor changes which have led to a huge change of direction. In the next section, I will recount how we have managed this change.

Implementing change at our school: beginning the process

Three years ago we identified maths as an area for development. While other local schools were following one direction in maths, our staff did not believe it was a suitable direction for our school setting. However, within the school there was a noticeable lack of common practice and in my role as co-ordinator, I had concerns that the standards of mathematics teaching were slipping. We needed to both find a new way forward and allow time for development.

Involving staff

All teaching staff were involved in discussion of the need for change from the beginning. Therefore there was a shared belief that we needed to do something to improve our standards and a common awareness of why change was being implemented.

Consequently the changes implemented were an identified need that was shared and discussed with staff rather than a random change of direction. This was an important factor in the success of the implementation process.

After investigating possible professional development options we established a link with CIMT. The research-based nature of the programme and overriding theme to move primary maths to be more mathematical with an emphasis on logic and rigour appealed to us.

As a leadership team, we looked at the 20-week course that CIMT offered and spent time discussing if it was the way forward for us. There were three units:

- Effective maths teaching
- Teaching mathematical foundations
- The collaborative practice model for enhancing mathematics teaching

We considered the pros and cons in relation to our staff, our school, where we were at that moment and where we wanted to get to.

Importance of time

Time is an important factor in implementing change. Evaluating whether something is right for your school and predicting how individuals will respond is time well spent. As a co-ordinator, this will have a greater impact if you can work it through with another member of the leadership team. Collaboration between members of the leadership team requires time, but it also leads to more sustainable change. Taking time to identify staff who are likely to be resistant to change and creating opportunities to undertake collaborative work with them can make a big difference.

We decided that the two-term course would enable us to stop and reflect, so that at the end of it we could make a more informed decision on the best way forward. I presented this to staff, explaining that the two terms would give us valuable time to reflect on our own practice, explore good practice from other countries, reflect on research and try things out. Having been inspired by Professor Burghes, the staff were keen to be involved with CIMT and so our gradual shift in direction began.

Staff motivation

'Hooking in' staff with inspirational speakers, writing or recordings of good practice reaps benefits. If staff are interested, involved and motivated, they will be more open to change.

The first two terms

We were fortunate that Professor Burghes visited the school each half term during these early stages of change. The staff really responded to his wealth of experience and his belief in the fundamental principles that:

- spoken and written maths should always be precise and use correct notation, definitions, language and layout
- all children can become mathematical thinkers
- we need to provide an integrated approach to maths, with numeracy being developed as part of maths.

There was an underlying feeling that he would give us all the answers and we were initially quite frustrated that he didn't! Looking back, this was empowering. At no stage were we told what to do or what to think; rather the approach was for us to try things and use resources that enabled us to stop and reflect. I set up meetings and introduced materials in a way that got people talking and then encouraged them to try things out and feed back at the next meeting. Staff had the time and flexibility to try things out in their own classroom without the pressure of 'now you must do this'. Some staff made changes to their practice immediately, whilst others stood back initially before tweaking their existing practice. In this way, staff had control – the change wasn't imposed, they were free to try things at their own speed. Looking back, I can see this was vital.

In the first two terms, we didn't introduce new teaching resources. Professor Burghes asked us to think about seating, active listening and engagement levels. We were encouraged to try seating children in rows for maths, with children paired so that they could learn from each other. He also suggested that our daily sessions should be more interactive, with maybe six short units in one session. There was an emphasis on the importance of children practising by doing, teaching each other and making immediate use of their learning.

Staff took these ideas and adapted their existing lessons accordingly as they saw fit. Again there was no pressure, although at this point there was the expectation that they would give something a go.

As well as modifying our existing practice, we also did background reading and watched examples of good practice from other countries. As the co-ordinator, I made sure that all staff had time to talk about what they had seen and read. I collated everyone's thoughts, both the positives and the negatives, and gave each member of staff a copy so that we could reflect on these and try out different things before our next meeting.

Involving all staff

In our meetings, we regularly use the tools that we use in our own classrooms to gather feedback. For example, as a staff we sit in groups and record all our ideas on a large sheet of paper or we go around the circle and everyone has a chance to speak. This ensures that everyone has a voice and can express an opinion, not just the vocal minority.

The reading and videos took us beyond our school and really made us reflect, question and want to try things out. Staff became very passionate about maths as they challenged ideas, encountered problems, found solutions and celebrated the impact that minor changes were having within their classrooms. As a staff we were buzzing and the gradual process of change was beginning to create its own momentum. There was maths talk everywhere… the staffroom, the playground, the corridors:

'I've just tried this…'
'The children are…'
'How do you manage…?'

As the maths co-ordinator, I couldn't go anywhere without being caught in a maths conversation, but most importantly, the staff were all talking maths with each other too.

By looking beyond our classrooms at what constitutes effective maths teaching, reflecting and then having the time to try things out and gradually adapt existing practice, we had raised the profile of maths across the whole school within a very short space of time.

Collaborative practice (lesson study)

Towards the end of the second term we began our collaborative practice sessions. Undoubtedly this was the most successful element of the process of change and something which the staff found extremely useful.

We all know the cliché that the school's most valuable resource is the staff, but how often do we actually go into each other's classrooms and watch each other teach outside of a monitoring and evaluation role?

We all make assumptions about what happens in different year groups, but how do all the staff (not just the leadership team) know what is happening in other classrooms if they have not been into them? Collaborative practice gave us just such an opportunity.

The collaborative practice model

- A group of teachers plan, observe and evaluate lessons.
- Observers make notes on agreed focus, good points of the lesson and what could be improved. The focus underpinning everything should be pupil learning.
- The review of the lesson is led by the chairperson, who changes each time. The teacher who taught the lesson starts, followed by the other observers in turn. Everyone starts with the positives, followed by open discussion.
- Action points relating to effective mathematics teaching are agreed. This should enable participants to improve their own practice and to help others.

Planning the structure of the collaborative practice was vital to its success. In my mind at this point, I had a clear sense of direction and I was able to anticipate and map the potential barriers that stop initiatives in their tracks and prevent them moving through towards sustainability.

For many members of staff, being observed by colleagues in this way was new and this had the potential to make them feel vulnerable. I spent a lot of time planning my groups for the first cycle so that we got the most out of the experience. Above all, I wanted to make sure that everyone would feel secure within their group. We had four groups and I ensured that each group had someone from Foundation, KS1, lower KS2 and upper KS2 so that we all saw the bigger picture. I put the more vocal staff together and considered how particular staff could be developed by working alongside specific colleagues. The headteacher and two of our teaching assistants also joined the teams.

A difficult aspect of collaborative practice is the organisation, as finding a clear week in the school diary is always a challenge! It helps to set the dates at the beginning of the year. Although there may need to be shifts, it is important that the leadership team agrees on the value of implementing the collaborative practice cycle.

We set aside two days each half term, initially within one week but then in consecutive weeks. We managed staff release by doubling up classes and using teaching assistants. Although this was not ideal, the benefits gained were immense. We used assembly times for planning the sessions and staff gave up their lunchtimes to evaluate the sessions. The staff meeting in the week following collaborative practice was set aside for maths and this gave us the opportunity to come back together as a staff so that we could keep moving forward together.

We found collaborative practice to be beneficial in the following ways:

- Teachers' subject knowledge improved.
- There was an understanding of the bigger picture: how children progressed from Nursery to Year 6.
- We developed a greater understanding of how we all worked.
- Staff saw good practice in each other's classrooms and adapted it to their own year group.
- There was an increased sense of collegiality.
- It had a positive impact on the ethos of the school.
- It generated lots of discussion.
- It focused and challenged our thinking.

Introducing MEP

Towards the end of the second term, we made the decision to follow the Mathematics Enhancement Programme (MEP). We had begun our cycles of collaborative practice and had had several staff meetings focusing on effective maths teaching. Practice had already shifted considerably and staff were positive about the changes, whilst also acknowledging issues.

We made the decision to use MEP together and this comes back to my earlier point: adopting MEP was something that the staff decided to do rather than it being imposed upon them. It was a natural progression and another slight change of direction. The whole staff gave their input to this decision. We went around the meeting, with every member of staff expressing their opinion, both positives and concerns before deciding to adopt the programme. Further discussion from the staff clarified that in order to assess its impact we would:

- follow the scripts with rigour
- use MEP for at least two years
- continue the cycle of collaborative practice and staff meetings.

We decided to use the IPMA tests (international yearly tests provided by CIMT) to give us an idea of the starting point for each year group, and each KS2 class began by using the material from the year group below. A lot of discussion was centred on ability-grouping the year groups into class sets, but the final decision was to use heterogeneous year groups. We also decided that we would provide additional teaching assistant support for children who were struggling.

Starting the programme was undeniably hard. There were lots of prerequisite skills and language that the children needed and in the first term nobody managed to get through an entire lesson plan in the space of an hour. Despite our earlier work, following the scripts was quite a significant shift in direction and both staff and children hit highs and lows in the early stages. For the children, the style of the lessons was quite different. They were being asked to share their ideas more regularly and had to engage in sustained shared thinking at a much higher level than in previous lessons. For the staff, following a script was quite demanding as was keeping the whole class focused and actively listening. At the upper end of the school in particular, teachers' own mathematical knowledge was being challenged and this increased the amount of time that they spent on planning.

There were several things that kept us going:

- Primarily, staff believed in what they were doing. They acknowledged that it was hard but they could see the point of what they were doing. They had been part of the decision-making process.
- We could see the positive impact that it was having on the children's learning: not only in terms of their mathematical attainment but also in their transferable skills beyond maths. The children were constantly surprising us with their achievements.
- Collaborative practice gave us a real sense of collegiality. Everyone was in this together and people were talking about maths everywhere. We were all hitting highs and lows at different times and in this way we supported and inspired each other. People were sharing their experiences and expertise with each other. We were all learning together.
- Regular staff meetings buoyed us along and kept us together. When change is introduced, doubt and resistance are natural reactions and it is likely that without these regular meetings the pockets of doubt would have spread. This could well have resulted in a factional staff, with MEP working well in some classes and not in others. As the co-ordinator, I needed to be very aware of what the issues were and where the doubt was likely to come from. It's about knowing your staff and working with them in such a way that they remain engaged in the process.

Staff meetings

It is important to prioritise regular meetings and as a co-ordinator, you need to work with the leadership team to map your staff meeting dates for the year. We had about one staff meeting per half term.

The focus of the meetings was chosen through informal feedback from the staff. Everyone was talking about maths so much at this point that I was very aware not only of where things were going well but also of concerns that were arising. In our meetings I aimed to:

- have a clear objective that addressed a current issue
- give everyone a voice and an opportunity to be listened to
- always begin with the positives – *this boosted morale and for the doubters and those resistant to change, listening to a stream of positives from their colleagues and identifying a positive themselves are helpful*
- give staff the opportunity to air concerns and identify issues – *with a view to addressing these in the future so that they didn't become a list of excuses*
- facilitate the meeting so that staff could work together towards solutions – *drawing on the expertise and professionalism of all the staff to drive things forward.*

Tools for sharing ideas in staff meetings

Giving everyone a voice in staff meetings was an important part of the process of change. Our staff are familiar with the critical skills tools and we regularly use these for sharing ideas. For those of you who are unfamiliar with them, I will briefly describe them here.

- **Sweep** – going around in a circle so that each member of staff has a turn to speak without being interrupted.
- **Brainstorm** – staff sit in small groups. Each group has a large sheet of paper and one person records all of the ideas with a chunky felt-tip. The focus of the brainstorm is written on the paper. It is important that every idea is written down, so that even the quietest member of staff gets heard. At the end of the allotted time, the ideas can be distilled down by highlighting the three most important points raised. These can then be shared with the rest of the staff.
- **Carousel** – this is a useful tool for getting a lot of feedback in a short time. The staff are divided into about four or five groups. Each group has a different coloured pen and a different starting point / question written on their large piece of paper. Each group spends a few minutes brainstorming their ideas before moving on to the next station, taking their pen with them. Everyone moves at the same time and when they arrive at each station, they tick any of the ideas that they agree with before adding their own. When everyone is back to their original place, each group distils the ideas on their original sheet before feeding back to the other groups.

An example of a carousel, with the three final points raised, at the end of the process:

How can we help our least confident children develop as successful learners? What is it that is holding these children back?

- Confidence
- Numeracy skills
- Listening and language skills

What key mathematical skills do these children need to move forward?

- Practical skills
- Mathematical language
- Key concepts

What messages do these children need to move forward? (verbal / non-verbal, peers / adults)

- That you value them trying
- It's good to feel challenged
- That they must and can do some things independently

What key learning behaviours do they need to be taught and reminded of regularly?

- Positive attitude 'I can' (at their level)
- Verbalising
- Actively listening

What strategies can we use in our whole-class lessons, so that these children make the transition from 'learned helplessness' to seeing themselves as 'I can' learners?

- Target them with questions they can answer
- Show confidence in them: raise their expectations of what they can do
- Preparation: if possible, practical experience before the lesson

After our first half term of teaching MEP, our staff meeting had the following structure:

- Positives of MEP
- Strategies that we have used to engage learners in MEP lessons
- Issues that are yet to be resolved.

Excerpts from staff meeting notes after using MEP for half a term: the ideas were generated by the staff using the brainstorming technique. There were many more!

Positives of MEP

- The children's development and use of mathematical language are significant
- The children are asking mathematical questions and are coming to the front to explain their reasoning
- Professional development – we're getting better at maths too, it's raising our understanding
- Children are thinking more logically
- Lots of revisiting – if they don't get it this time, it comes round lots of times
- More direct teaching – less opportunity for drift
- Problem solving is less of an issue for them, they approach it logically
- Children are being stretched and they are enjoying the challenge

Strategies that we have used to engage learners in MEP lessons

- Taking the time to establish rules, routines, expectations of listening etc
- Planning ahead: giving children active/concrete experiences prior to the lesson
- Modelling – teacher as the learner
- Using physical activities, songs and action games as brain breaks
- Use of exercise books for open-ended activities to extend/consolidate learning
- Getting less confident children out to the front early in the lesson and giving them praise
- Changing places/partnerships – particularly if children are anxious or not engaged

Issues that are yet to be solved

- Differentiation – meeting the extremes
- Foundation / Year 1: meeting the needs of child development

Change is a difficult process; therefore it was important for morale to identify the positives as well as the issues to be solved. It was also important to keep a focus on the principles of effective maths teaching. The impact of whole-class interactive teaching would be lost if children were not focused or engaged in their learning and this was something that staff needed to reflect on so that they could then take ideas back to their classrooms. By using the brainstorming technique, staff generated lots of ideas for engaging learners in MEP lessons and these sparked a really good discussion. It was much more powerful than providing staff with a list of things to try. At the end of the meeting everyone's ideas were collated and distributed to be used as an aide memoire.

By the beginning of the second term, both the staff and the students were feeling more confident with MEP. We organised a meeting for parents so that they had a greater understanding of what we were doing and could ask questions. We wanted the parents to have a clear insight into why we were following this programme, and so we asked Professor Burghes to run the meeting. The parents received his presentation very well and this helped the process of change. We decided to hold this meeting after a term of settling in so that everyone, including the parents, had had time to become familiar with the change and the impact that it was having on the children.

The process of change is ongoing and it would be wrong to think that by the end of the first year all was well established. Since then we have continued to identify positives as well as issues and to explore solutions. Other starting points for discussion have included:

- How is MEP helping children to succeed as learners?
- What do you think the children are getting better at?

Prompts like this at the beginning of staff meetings have generated lots of positive comments and this has made the staff even more determined to find solutions to the issues.

Differentiation and inclusion are issues that are frequently raised when we talk about whole-class teaching and we frequently come back to finding solutions to these issues in our conversations.

The tight spiral nature of the programme in itself supports differentiation, as regular revisiting gives children time to assimilate new ideas in different contexts. We soon realised that MEP was very cleverly written, in that what appeared to be totally random activities were actually drip-feeding new thinking into future learning.

Children's partnerships

The children's partnerships are also vital in creating an inclusive classroom, and teachers need to spend time getting these right. It doesn't work if all the confident mathematicians are simply paired up with those who are struggling. This may give the teacher a quiet life, but in our experience the more able child does all the thinking or gets bored, whilst the less able child stops thinking altogether and just writes down what they are told. This is not inclusive practice and as a co-ordinator it is important to lead staff in reflecting on the impact of different pairings.

Within our experience, we have found that children gain the most from the collaborative paired work if the difference in their achievement is not too great – although there are some occasions when it suits the needs of the learners to have a wider gap. We have also found that it is important to swap the partnerships over so that a child who is the lower-achieving student in one partnership then works in a partnership where they are the higher-achieving student. By working in pairs and fours on a problem, children are supporting and extending each other and making immediate use of their learning. Teachers must be aware of children's needs by circulating around the class and listening to their responses. This is more powerful if teachers haven't over-coached children through the script but have let them think for themselves about how to approach a problem. By using formative assessment, they can then target their questions to support or extend. Carefully selecting children to explain their reasoning at the front of the class offers further opportunities to support learning and to encourage children to challenge each other's thinking by asking questions. It is also a good opportunity to show how problems can be represented in different ways – linking the concrete with the abstract and moving towards generalisation.

The above points have come from the staff's experiences. As a co-ordinator, I was very aware that the role of the teacher was crucial in developing an inclusive whole-class approach and many of our starting points for discussion have been chosen with this in mind. For example:

- *Think of a time when you have struggled in a learning situation… or of a child who has suddenly grasped something. What strategies helped?*

Staff recorded their responses on sticky notes and these were stuck on a life-size cut-out of a child, which was left up in the staffroom for a while before being collated. The list of ideas that staff generated was vast and they were then asked to reflect on these in the light of their own classroom practice.

In a staff meeting at a later date, staff were asked to imagine that they were a child who struggled with maths. They were given three different scenarios to brainstorm:

- *You struggle in maths every day… you don't think you will ever be able to do it… what strategies do you adopt to get you through the lesson?*
- *This year, it's worse. What is happening in your classroom that's making it hard for you to learn, hard for you to take part?*
- *You're in a new class… you are beginning to believe in yourself… you can do maths… you're not as good as the others but you are learning and everyone is proud of you. What's different? What is the teacher doing that is helping you to learn?*

The aim of this meeting was to consider the role of the teacher in developing a classroom culture which supports all children as learners. Similar starting points could be used to generate ideas for extending more able pupils.

By working in this way, all staff are able to share their knowledge and influence each other's thinking. If you have less confident staff or staff with professional development needs, this is a powerful approach which is more beneficial than an 'expert teacher' coming in and telling them what they're doing wrong and how they need to change.

Other issues and solutions

An initial problem was the implementation of MEP in Foundation as there were no materials for this. However, our nursery teacher was passionate about laying firm foundations for successful learning and took the principles behind MEP and adapted the themes from the Reception book to create engaging learning experiences for our youngest children. This was a gradual process of trying things out, finding what worked and adjusting accordingly. Collaborative practice has been very useful for getting staff from across the school into Nursery to see what these young children can achieve and also to recognise that there are common issues from Nursery to Year 6.

Staff in Reception and Year 1 followed the materials as they were written for the first year but soon realised that they needed to be adapted to make the learning more meaningful and active for their young learners. They have provided more concrete experiences and have spent time developing mathematical thinking and language. In Year 1, written recording has been a difficult issue along with the abstract nature of some of the material in the books which can be difficult for children to visually comprehend. There is undoubtedly still a lot of work to do here, but staff have not given up on MEP as they have an understanding of the bigger picture. They have worked together to find solutions and adapt their practice, focusing on the needs of the learner. It has been safe for staff to challenge and question whilst still maintaining the shared belief in the principles behind MEP.

We are constantly looking at the needs of the children and there is close collaboration between myself as co-ordinator, the class teachers and the SEN team to discuss appropriate support for children. We have two teaching assistants who provide children with additional maths support. For children who are really struggling to keep up with their class, this small group support is given during the maths session. Other children receive support in booster sessions outside of the maths hour, mainly for numeracy.

Conclusion

Three years ago we embarked on a gradual process of change that has had a significant impact not only on the teaching and learning of maths at our school but also on the ethos of the school as a whole. This change has been led by a shared belief which is driven by values. We have had a common goal and the work with Professor Burghes has given us a clear sense of direction.

As teachers within a professional community we have worked together, sharing our ideas and challenging our thinking. Decisions for change have been based on shared knowledge and collaboration. The process has required us to be open to ideas, to be honest and to reflect on our teaching. Teachers have learned from and with each other. They have worked collaboratively to address issues and have benefited from planning and observing lessons across the school.

Our journey has been one of minor changes leading to a huge shift. The challenge that now lies ahead is one of sustainability: the danger of drift as other subjects take precedence.

As a co-ordinator, I need to maintain the process of change and focus on the fundamental principles of effective mathematics teaching. Monitoring and evaluating, looking at assessment and standards, working alongside colleagues by partnering and keeping maths as a focus with the leadership team will all help to keep the momentum going in the future.

Change is exciting when you can see the positive impact on the children and the school. As professionals, that is what we are most influenced by. You know when something is working and this underlying belief helps drive change forward, through stages of adjustment and doubt towards sustainability.

5.3 Starting in nursery

Sharon Kellett, *Grouville School, Jersey*

Background

I am the nursery teacher and Foundation Stage co-ordinator at Grouville Primary School in Jersey. Our nursery provides 30 places to children aged three years and upwards, of whom the majority go on to the Reception classes at our school. The nursery team consists of a teacher and two nursery officers. We are a well resourced nursery with ample space both inside and out. Our team is enthusiastic about new initiatives and are willing to take risks. This is our journey so far…

Why did we begin using MEP?

In September 2007, the whole school was invited to an INSET course entitled 'To move primary mathematics to be more mathematical' provided by Professor David Burghes. With a view of myself as a non-mathematician and holding an attitude of fear towards mathematics, I approached the course with reluctance. However, this course ignited my motivation for teaching mathematics and I was dismayed to find MEP did not include nursery!

It is unfortunate that often whole-school initiatives take place without nursery provision, leading to feelings of isolation and a lack of belonging. The importance of continuity of care and the need for firm foundations being started in our nursery class led to my initiative to generate pre-Reception ideas and adapt the set of Reception MEP resources.

How did the training support my professional development?

The course was in three parts:

- Effective maths teaching
- Collaborative practice
- Foundations of mathematics

A foundation of the MEP philosophy is that 'all children can become mathematical thinkers – they can all think mathematically'. This statement resonated with me and encouraged reflection on my own practice, culminating in the realisation that my mathematics teaching in nursery focused on numeracy and shape and neglected the important ideas of logic, understanding and application of pure mathematics.

An initial step at our school was to develop a collaborative practice team to work with Professor Burghes. This consisted of representatives from each phase of the school who were both confident and enthusiastic to lead the rest of the school. As a team, we examined the materials, jointly planned lessons for each of us to deliver and set up focuses for observations. Each team member then delivered the lesson whilst the rest of us observed. Engaging in collaborative planning and observational feedback cycles were both crucial to the initial success of MEP through developing our mathematical knowledge and knowledge of teaching mathematics. Observing lessons across a range of age groups provided us with the understanding of how the material could be used to empower our children and challenge them to think. Within the nursery context, my colleagues commented positively on the engagement and responsiveness of the younger children.

The starting point in nursery

Our nursery teaching philosophy draws on a child-led approach to learning and play. My initial investigation of the Reception materials left me slightly apprehensive as the MEP structure was for whole-class teaching and left little opportunity for child-initiated learning. Therefore I began by trialling the Reception materials as presented and attempted to engage the young children in large group lessons through cultivating an approach which included an appropriate pace and focus intertwined with an emphasis on active learning.

During the first trial year I dipped into the topics covered in the book, then took out the simplest bits and expanded on them with a range of visual and experiential activities. For example, I adapted an activity called 'Meet the family – Using the three bears' family'. In these lessons we focused on ensuring that the family members terminology was understood: counting the three bears, comparing sizes, developing use of the terms *big*, *medium* and *small* and ordering the family in different ways. These would be familiar practices to many nursery practitioners and teaching in such a way reflected a different context of teaching rather than different teaching for me. The key change was the emphasis in MEP on the accurate use of mathematical terms from an early age with the expectation that children are encouraged to come forward and verbally contribute to sessions through consistently explaining their answers. In addition to the topic-focused work, we would incorporate active counting, games and songs into our lessons. Initially, the children were thoroughly engaged for the planned 10 minute slots; however, this quickly increased to 20 minutes as their interest levels and enjoyment of mathematics developed.

As my confidence in using and adapting the MEP material grew, it became very evident that this style of teaching with very young children would only be successful if I was able to manage the children's behaviour in such a way that promoted and embedded the key dispositions for learning. The teaching style I developed through adapting and using the MEP resources supported children's development of key dispositions such as sitting still, focusing and responding to questions. During this first year I produced assessment sheets for each child based on the dispositions they needed to access the MEP teaching style and concepts covered in the ten MEP topics developed in the Reception book. The topic assessment sheets were onerous and unhelpful; however, the dispositions assessment sheets have continued to be an invaluable resource.

How did we adapt the MEP resources in our second year?

Following the first trial year of using the adapted Reception materials, it was decided to review the content for a number of key reasons:

- Our curriculum planning is underpinned by the Early Learning Goals and it was felt that the MEP material had gaps which needed to be filled in order to meet the needs of the Early Learning Goals.
- The majority of our children proceed to Reception classes within our school and we did not want them to repeat the work.
- The use of posters was not the most effective way to present the concepts to the children.

To address the needs, it was decided that use of real-life situations in the home corner, outside in the garden or in the construction area provided more powerful learning experiences for the young children. Using sand, paint and corn silk were better vehicles for teaching hand/eye co-ordination and numeral formation than photocopied sheets or interactive whiteboards. The changes we implemented were possible through reflecting on the needs of our school context and learners and the underlying philosophy of the MEP material.

Following this review, plans were established for the long and medium term which integrated changes to our delivery such as use of real-life situations to meet the needs of our children, and took account of the perceived gaps between MEP material and the Early Learning Goals. Therefore a bank of ideas was developed alongside a syllabus that was tailored to the needs of our school and learners which covered and consolidated the key concepts to be delivered in the Foundation Stage.

Learning through engaging in collaborative practice

For my personal development in implementing MEP at Grouville School, collaborative practice played a vital role. Initially, it helped me begin to understand the MEP programme itself. Later it supported the growth of my confidence to chair my own collaborative team in a curriculum area in which previously I had lacked confidence. The success of the collaborative practice model which we developed was due to a number of reasons:

- Teams were well balanced and members reflected a range of teachers of differing year groups within the school.
- Observing groups twice across the age groups allowed us to see how the MEP material spirals and develops and challenges the children's thinking.
- The two rotations of planning, teaching and reviewing were enjoyable and informative. Joint planning meant shared responsibility. A common focus for observations highlighted areas of concern and allowed us to work as a team to discuss problems and seek solutions.

The personal benefits of engaging in collaborative practice were that it highlighted our achievements within the nursery class and provided opportunities for other teaching staff to observe the capability and achievement of the nursery children. Previously many of my colleagues had never been in the nursery and therefore this provided them with opportunities to develop their understanding of the learning which takes place in the class. Additionally, the opportunities to observe children in different year groups across the school highlighted that in other classes there were similar problems to those I faced, which were often related to children's development of the dispositions needed to be a good learner.

How have the children responded to MEP?

We aim to include three 20-minute mathematics lessons a week and the class shows high levels of enjoyment in these. Timetabling an early slot in the day ensures the children are fresh and eager to learn. The lesson begins by gathering in the maths circle and undertaking an active warm-up such as jumping in the circle, counting aloud, and throwing large dice. Following the warm-up, the children move to the carpeted area where I have my maths board and where the focused teaching work takes place. The children love this time and are enthusiastic to be chosen to come up to the board and work. This can involve practices such as answering questions, pointing out features, physically touching material and counting independently and publicly, and I believe it supports the development of confident mathematicians. The children are encouraged to have a go, they are also invited to help and give guidance if it is needed. It is necessary to closely monitor who is chosen to come forward during each session to ensure no-one feels left out and also because the children who do not get an independent turn each session can find it difficult since being an active learner has become so important to them. The focus of the lesson varies between being child-initiated or focused on a specific objective. The most successful sessions are the ones where the children take some ownership of the session. After the focus, we always end with a song often involving props with the children up on their feet. A 10-minute focus is a long time for some of the children and they love to get up and move after all their hard work and find it rewarding.

Parents' and carers' response to MEP

Many of our parents actively prepare their child for nursery, encouraging them to be independent, fostering a love of books, and teaching them to write their name. However, very few appear to prepare them mathematically other than counting with them. Often our parents are surprised at their child's love of numbers. During parents' week we hold sharing practice and last year this focused on mathematics. However, it was unusually poorly attended by parents of children in nursery. Those who did attend expressed surprise both at our expectations of the children and their own child's mathematical capabilities. Response sheets which were gathered following this sharing practice suggested that many parents had negative perceptions of mathematics and were uncomfortable in supporting their children to learn mathematics, viewing it as a subject better left to be taught and developed at school.

Further development needs

It is our third year of implementing MEP and in nursery we are satisfied with both the planning and structure of the lessons. With each cohort, it is evident that whilst the intensive 10–20 minute sessions are enjoyable, there are inevitably some children who are not accessing the concepts or are not joining in as well as they might. Consequently we are addressing this by introducing mathematics time outside of the planned lesson and developing our use of table-top mathematics, independent work at the maths board and mathematics outdoors.

Using a whole-class teaching approach in nursery

Fundamental to the success of the whole-class teaching approach is positive behaviour management. To ensure this, we involve the whole team as part of every lesson and as adults we strategically place ourselves to ensure the children who find it difficult to sit are monitored and the reluctant participants are encouraged.

An essential aspect to ensure whole-class participation is the use of a written log to track those who have not been up to the board or answered questions during the session.

Within our class, it is also necessary that we provide support for some of the children with learning needs such as communication issues and EAL learners. In one case, the specific points need to be signed to a child and her understanding checked through a one-to-one interaction. Similar practices need to be undertaken with our EAL pupils; therefore it is important that time is taken after each session to ensure individual needs are being met. The maths table is used to ensure that this is achieved.

What does MEP look like in the nursery class?

A typical lesson in my nursery class would usually contain the following elements:

- an active warm-up
- a central theme (adapted from the planning)
- a game or song.

A lesson takes place on three mornings per week and initially lasts between 10 and 15 minutes. Following the lesson, the materials are left on the maths board for children to use and revisit and a maths table is always set up with resources that will allow the children to apply what they have learned.

Mathematics will also be encouraged outdoors, in the creative area, in the construction area and in the sand and water trays.

Active warm-up ideas

- **Count and Clap** – The children are asked to count aloud whilst on their feet in the maths circle. Children may be following instructions from the teacher, a number card or from other children.
- **Jump To It** – A dice is thrown in the maths circle. A child is asked to count the spots and we all jump and chant that number.
- **Roll It** – A dice is rolled across the maths circle to another child who counts the spots and rolls on to another child calling their name.
- **Pass It** – A ball or toy is passed around the circle and children have to say, for example, a colour or a shape.

Some central themes (see Appendix on page 70 for complete list of topics)

It's all about creativity!

Topic 1 – Family life was taught through the story of the three bears. Counting to three, and the terms *big*, *medium* and *small* meant the maths focus became an integral part of this learning. Role play and dressing up engaged the children to further enhance their learning. They played in threes, they used mathematical language and they solved mathematical problems.

Topic 2 – Positional language gave rise to 'The position challenge' where we photographed groups of children in the garden with various resources and asked such questions as: Who is in the picture? How many children are there? What is Emma holding? What is beside Henry? Who has the blue bucket? How many buckets are there?

Topic 3 – Night and day took us into space where we visited the red planet. On our rocket we counted to 5 and back before blast-off. We counted stars on our journey.

We found aliens, described their features – for example three eyes, two legs etc. and named them according to their colour.

Starry fingers were used to learn about space – high, low, left and right.

Table-top activities were rocket-building, colour-matching games, matching aliens.

Topic 10 – Transport asked us to look at cars, maps and orientation. We counted cars and discussed the rules of the road. A 3D map made by the children was then used as a play-mat to reinforce the rules of the road and gave us opportunities for lots of directional language. Car-washing, building a toy car from scratch and a trip to a car park gave rise to lots of new associated vocabulary and increased the levels of motivation.

Games and songs

Suggested games

- **Team games** – These encourage turn-taking, excitement, logical thinking and speed.
- **Team colours** – Children in turn collect a card of their team's colour. Children sit in a line, collect a card, and join the back of the line. The team to finish first are the winners (this involves learning about colours and one-to-one matching).
- **Find my babies** – Each team has a mummy insect looking for her 10 babies. Using a similar process as above, however, the children collect smaller versions of the same insect and check how many they have found (this involves learning about team work, one-to-one matching and skills in counting to 10).
- **Photo lotto** – Collect a photo of everyone in your team (this involves learning the names of all team members and developing observation skills).

Song time
We use songs from various number and counting CDs as well as songs we teach with props. These include:

- *Five green and speckled frogs*
- *Ten green bottles*
- *Three little ducks*
- *Farmer in his den*
- *Heads, shoulders, knees and toes*
- *Three spacemen in a flying saucer*
- *Five little monkeys jumping on the bed*

The songs incorporate a lot of actions and using props encourages children who are reluctant to participate as they all want to hold a prop even when reluctant to sing.

How do you resource MEP?
Initially I was provided with a Reception handbook and poster pack. Over time we have collected resources for each topic. During the lessons, visual aids and pictures from clip-art are invaluable for setting the scene. We also incorporate number cards, dice and real-life apparatus (within their experiences). Some of the topics can be taught through experiential activities, for example in Topic 5 the routines can be taught in the home corner or the kitchen at snack time, and for Topic 6 (with a focus on seasons) we are able to teach this outside and through our Forest School programme. This has supported the effectiveness of implementing MEP through integrating real-life aspects and focusing on the skills young children need to develop to become good learners.

I believe nursery MEP is about equipping the children with the dispositions to become good learners, inspiring them as mathematicians, questioning and challenging them to be logical thinkers and creating a safe environment where they can be confident and excel.

Appendix
MEP adaptations for the nursery class
A year at a glance

Topic 1 My family and friends *Autumn Term*

Family – names and ages
My home – spaces in the house
Drawing faces
My friends – community building and group dynamics
Rote counting to 5
Colours and fruits

Topic 2 Around our school – locations

Positional language
Autumn theme
Counting to 5 in games

Topic 3 Myself

Life at home and special times
Parts of the body
Celebrations
Night and day – light and dark
Counting out five things

Topic 4 The weather

Winter – rain and snow
Water – capacity and pouring
Orientation in space
Number 6

Topic 5 Routines *Spring Term*

At home and at school
Eating and table manners
Menus – café and laying the table
Sleeping time
Number 7

Topic 6 The seasons

In the garden

Spring and Easter

Clothes for all seasons

Number 8

Topic 7 Animals around the world

Developing comprehension and oral skills

Orientation in space

Left and right

Number 9

Board game skills and puzzles

Topic 8 Shopping and shape *Summer Term*

Preparing for addition

Packaging

Patterns

Money and tills

Number 10

Topic 9 Developing writing

Counting and using numbers 0 to 10

Recognition of numbers

Beginning to trace, make and write numbers

Developing manual skills

Topic 10 Transport

Traffic and maps

Understand rules of the road

Safety – Green Cross Code

Developing comprehension skills

Understanding signs and responding to them

Spatial awareness

Using numbers in games and maths 0–10

MEP Learning Autumn Term Nursery Class Adaptation

These activities are extra and correlate with the activities in the book (Topics 1–3 Reception MEP)

Topic	Learning	Activities
My family	Talking about family members	Look at photos Male and female Names Place in family e.g. mum son, etc Draw family
Friends	Observational skills	Getting to know who's who Friends bingo Community ball Name game
Colours	Matching skills	Match colours Naming colours Mouse paint challenge Colour balloon game
Fruits	Observational and manual skills	Looking at fruits Naming fruits Colours of fruit Counting fruits
Around the school	Following lines	Maps Train ride Directions game
Positions	Left and right positional vocabulary	Photo challenge Small world scenes
Autumn	Using a theme to understand position	Autumn pictures Colours Hibernation
Myself	Name body parts	Paintings Songs, games
Celebrations	Counting candles	Party games, celebrations

MEP Learning Spring Term Nursery Class Adaptation

These activities are extra and correlate with the activities in the book (Topics 4–6 Reception MEP)

Topic	Learning	Activities
The weather	Talk about the weather Clothes suited to weather types Living/non-living things	Looking at photos Weather forecasters Weather charts Dressing Teddy
Rain and snow	Observational skills Comprehension skills Short-term memory	Looking at pictures Kim's game Ski challenge
Water	Capacity Pouring Flowing Bubbling	Lots of water-tray play Pouring drinks Rain and puddles Bubble machine
Numbers 6, 7, 8	Understanding and using 6, 7, 8	Counting up to 6, 7, 8 Drawing sticks to illustrate Counting lots of things Ordering
Family routines	Days of the week Meal times Bed time Hygiene	Songs Home corner Laying the table Designing meals 5 a day
Café	Making a café Waitering and serving Orders	Home corner café Preparing snack Writing
Night-time	Time Darkness Need for sleep	Clocks and watches Songs Pyjama party 7 dwarfs
The seasons	The seasons Spring / New life Easter	Paintings Songs Easter activities
In the garden	Conditions for growth	Planting etc

MEP Learning Summer Term Nursery Class Adaptation

These activities are extra and correlate with the activities in the book (Topics 7–10 Reception MEP)

Topic	Learning	Activities
Animals around the world	Talking about animals in Jersey Animals in other countries Sizes and features Behaviours	Looking at photos Maps Zoo Small world
Shopping	Money Buying and selling Packaging	Home-corner Coins Shopping lists
Preparing to add	Concept of numbers and mathematical operations	Number play + sign Adding on One more
Shape and pattern	Noting differences between geometric shapes Naming shapes Drawing shapes Looking at patterns	Shape pictures Building pictures (houses) Shape lotto Computer work
Games	Rules of games Taking turns Using dice Moving around a board	Lots of games indoors and out Real board games Real-life games (Jump to it)
Developing writers	Observational skills Manual skills Fluency	Holding pencils; strong grip Playdough Preparing a snack Mark-making with accuracy
Transport	Traffic control Rules of the road Following routes	Bikes Green Cross Code Traffic game Maps

This will be used in conjunction with activities from MEP Reception.

MEP assessments

Dispositions	Autumn	Spring	Summer
Sitting nicely			
Eyes focused			
Joining in			
Listening			
Taking turns			
Coming to circle			
Responding to question			
Answering			
Questioning			
Copying			
Following instructions			
Sharing ideas			
Volunteering			
Hands up			
Thumb tool			
Having a go			
Concentration level (GSP)			
Physical skills (GSP)			
Attitude (GSP)			

5.4 Pupils' perspectives and attitudes towards learning mathematics following implementation of the Mathematics Enhancement Project

Jodie Hunter, *CIMT, Faculty of Health, Education and Society, University of Plymouth*

Introduction

Recently, there has been growing recognition of the need to investigate pupil perspectives and acknowledge their voice in order to understand schooling experiences (Boaler[59]; McCallum, Hargreaves & Gipps[60]; Morgan[61]). When whole schools undertake professional development such as that offered through the Mathematics Enhancement Project (MEP) and implement new ways of teaching, this can lead to significant changes in the classroom for pupils. As they are inducted into new ways of working mathematically, there are shifts in pupils' roles as learners. Bringing significant change in classrooms has ongoing consequences for both students' learning of mathematical ideas and for the development of their mathematical identity.

For many years, connections have been made between motivation and achievement. Consequently, when investigating how to enhance primary mathematics it is important to take into account the affective as well as the cognitive aspects of learning mathematics. Negative attitudes, anxiety and fear of mathematics as a curriculum subject have been well documented (e.g. Boaler[59]; Burns[62]; Wilson & Thornton[63]). This can impact negatively on children's achievement; therefore it is important to develop awareness of the pupils' perspectives on the teaching and learning of mathematics.

Current educational reform such as advocated by MEP calls for changes to mathematics teaching, with a greater emphasis being placed on language and communication. This involves children working together collaboratively and communicating their explanatory reasoning. Other key factors include mistakes being used as teaching points and an emphasis on developing mental strategies to solve problems. Developing these practices in the classroom can be challenging, particularly with regard to the beliefs about mathematics which participants may hold. For example, research by Young-Loveridge, Taylor and Hawera[64] found that pupil responses largely reflected a view that mathematics was a private, solitary activity in which talking to others was unacceptable. Furthermore, a number of pupils in their study voiced concerns that they may be seen as cheating if they took interest in their peers' solution strategies.

Research studies (e.g. Hodge[65]; Hunter[66]) have specifically analysed the influence of reforming classroom practices on student perception and identity in mathematics. Hodge investigated USA 1st and 2nd grade student perceptions with regard to their role and mathematical competence in two classrooms with significant differences in the forms of instruction. The students in the study spent 1st grade in a classroom with reform inquiry-based instruction methods, before moving to more traditionally-orientated instruction in 2nd grade. These students gave markedly different responses with regard to what it meant to be a good mathematics student in each class. In the classroom with inquiry-based practices, competence focused on talking, thinking and listening. In contrast, within the traditionally-orientated classroom, success was associated with steps and answers. Similarly, a New Zealand based study by Hunter investigated Year 5 and 6 students' views after being members of an inquiry classroom for one year in which explanation and justification of mathematical reasoning was the norm. Students in this study were able to identify a wide range of positive outcomes from engaging in the practices of listening to explaining mathematical reasoning. Also, they recognised the value of inquiry and debate as a tool to both analyse and reconstruct mathematical reasoning.

This section will outline the children's perspective of mathematics and mathematical learning in Grouville School, which has successfully implemented the Mathematics Enhancement Project.

Method

This paper reports on the views of 101 children from five classes at one school. The number of children at each year level is shown in the table below. The study included children from a range of ethnic backgrounds who were achieving at a variety of different levels for mathematics.

Data was collected from across the school including a written questionnaire with open-ended questions and other questions involving a Likert scale. Two versions of the questionnaire were provided, with one for pupils in Key Stage 1 and the second version for pupils in Key Stage 2. The results reported in this paper are from the questionnaires completed by the Key Stage 2 pupils. Five classes completed the questionnaire as shown on the table below:

Table 1: Number of pupils by year group

Year level	Number of classes	Number of pupils
Year 4	2	41
Year 5	2	44
Year 6	1	16

Results and discussion

This paper presents the views held by children from a school where the professional development associated with MEP was undertaken and the MEP curriculum material successfully implemented.

Table 2 overleaf presents the overall percentage of pupils who agreed or disagreed with each statement. The highest levels of agreement were on the statements about: the acceptability of making mistakes (98%); the importance of sense-making in mathematics (95%); persevering when work in mathematics is hard (94%); and the importance of being able to explain a solution strategy to the rest of the class (92%).

Table 2: Percentages (rounded to whole numbers) of students who agreed or disagreed with each statement.

	Strongly disagree	Disagree	Agree	Strongly agree
I am good at maths	2	17	66	15
Learning maths is mostly remembering facts and rules	7	28	55	10
I learn more by working with other children	3	18	48	31
I ask questions in class about maths	5	20	60	15
It is important to be able to explain how I solved a problem to other children in my class	4	4	46	46
Knowing why an answer is correct in maths is just as important as getting the right answer	5	22	41	32
When two children don't agree on an answer in maths they need to ask the teacher to see who is correct	24	35	25	16
I talk about my ideas in a group or with a partner	2	13	61	24
It is okay to make mistakes in maths	2	–	37	61
When my work in maths is hard I don't give up	4	2	44	50
When two children don't agree on an answer to a maths problem they can usually think through the problem together until they work out who is right and why	3	10	57	30
You can be good at maths without understanding it	37	35	23	5
I can come up with my own ways to solve maths problems	2	18	62	18
It is important for me to listen to how other children in my class solved a problem in maths	7	4	32	57
Adding, subtracting, multiplying and dividing are only a small part of maths	21	34	30	15
It is important to get the answer right in maths	26	48	18	8
Learning maths involves more thinking than remembering	5	18	60	17
Maths is difficult for me	17	47	25	11
Maths is interesting	9	14	38	39
Asking questions in maths means you didn't listen to the teacher well enough	35	50	10	5
When working on a maths problem it is important that your answer makes sense to you.	1	4	30	65

Disposition towards mathematics

The majority of pupils (81%) viewed themselves as competent mathematicians, agreeing with the statement 'I am good at maths'. Many also demonstrated a positive attitude towards mathematics as a subject, with 77% agreeing with the statement 'Maths is interesting'. For the majority of pupils (64%), maths was not viewed as personally difficult for them, and a large number of pupils (94%) reported that they did not give up when maths was hard. Many of the pupils were willing to ask questions in class, with 75% of pupils agreeing that they asked questions about maths.

Learning mathematics as a subject

Many of the pupils (65%) perceived learning mathematics as remembering facts and rules; interestingly, however, a greater majority of students (77%) also agreed that learning mathematics involved more thinking than remembering. The pupils placed greater emphasis on sense-making in mathematics than on getting the right answer. Mathematics was viewed as a subject area in which it was important that your answers made sense – with 95% of pupils agreeing that it was important that their answer made sense when working on a maths problem. However, only 26% of pupils agreed that it was important to get the right answer in mathematics and 98% of pupils agreed that it was acceptable to make mistakes in mathematics.

The pupil responses also indicated that many had a sense of autonomy when engaging in mathematics. For example, 80% of pupils agreed that they were able to come up with their own strategy solutions. The teacher was not viewed as the sole mathematical authority within the classroom, with 87% of pupils agreeing that when two children did not agree on an answer to a maths problem that they could think through the problem and work out who was right and why. However, pupil responses were more divided on whether the teacher needed to be consulted to decide which answer was correct when two pupils had differing outcomes – with 59% of the pupils disagreeing with this statement.

Learning mathematics was viewed as a social process, with 79% of pupils agreeing that they learned more when working with other pupils. In fact 85% of the pupils agreed that they talked about their ideas in maths with a partner or in a group. A large group of the pupils (92%) viewed it as important that they were able to explain how they solved maths problems to the class. Slightly fewer pupils (89%) agreed that it was important to listen to how other children in the class solved problems.

What is mathematics?

Pupils were asked to give a written response to the open-ended question *What is mathematics?* Responses to this question were organised into common themes. The most frequent response referred to the content areas of mathematics. For example:

Mathematics is learning how to subtract numbers from others, add numbers, divide numbers and multiply numbers. Mathematics is also learning which numbers are odd and even as well as learning fractions and all your times-tables.

Maths is shapes, measurement, time and weight.

Another common response referred to the usefulness or utility of mathematics. Frequently the pupils related this to future jobs. For example:

Mathematics is something which can be very useful when you've got a job because if you've got a job as a person who works at a till and the till breaks down and there are no spare tills, you'll need to add the prices up and give people the right amount of change.

Mathematics is something that you learn so when you are grown up you will be able to work in a bank or add something up.

A number of pupils also made reference to processes such as learning, thinking, problem-solving, and questioning. For example:

Mathematics is a subject which you have to think about.

Mathematics is numbers and reasoning. It is also about logic and thinking.

Pupil advice to a new child in the class who wants to succeed
Pupils were also asked to write the advice that they would give to a new pupil in their class who wanted to succeed at mathematics. The responses to this question indicated their values and beliefs in relation to mathematics. Many of the responses highlighted the need for effort and persistence, for example:

To always try their best at everything they do in a maths lesson.

Additionally some pupils referred to their belief that it was okay to make mistakes as these could be used as learning opportunities:

I'd say to them don't worry if you make a mistake, because that means you'll get it right next time.

It doesn't matter whether your answer is right or wrong, but if it is right, great. Basically knowing where you went wrong is better than not knowing.

Other responses referred to the social nature of mathematics in the classroom along with the need to ask questions:

I would tell them 'If you don't understand, talk to your partner'.

If you don't understand something, then ask.

The need to sense-make in mathematics was also highlighted by the pupil responses:

Think about your answer and make sure your answer makes sense.

I would say that when you are working on a maths problem, it is important that the answer makes sense to you. It's not the answer that matters, it's that you understand how you got the answer.

Discussion and conclusion

Overall the pupils generally displayed a positive attitude towards mathematics. Many viewed the subject as interesting and one which was worth persevering with when they were challenged by a task. They viewed themselves as autonomous mathematicians who could develop their own strategies and analyse solutions to verify whether they were correct. Sense-making was highlighted as an important aspect of learning mathematics and solving problems.

Reform in mathematics education such as advocated by MEP has placed a focus on collaboration between pupils and the importance of language and communication. Pupils in this study viewed mathematics learning as a social process. They valued opportunities to work with their peers and identified both sharing their own solution strategies and listening to others share their ideas as important aspects of learning mathematics. This contrasts with the results of research by Young-Loveridge, Taylor and Hawera[64], who found that many of the primary pupils in their study perceived mathematics as a solitary, private activity.

Mathematics as a subject was viewed by the pupils as useful, particularly for future work opportunities. Pupils made reference both to content areas and processes when asked to describe what mathematics was. Effort and perseverance were values which were perceived as strongly related to mathematical learning. Also emphasised was the need to use mistakes and errors as learning opportunities. Overall the pupils were positive towards learning mathematics and acknowledged the social aspects of this learning.

6 The role of the external consultant

Professor David Burghes, *CIMT, Faculty of Health, Education and Society, Plymouth University*

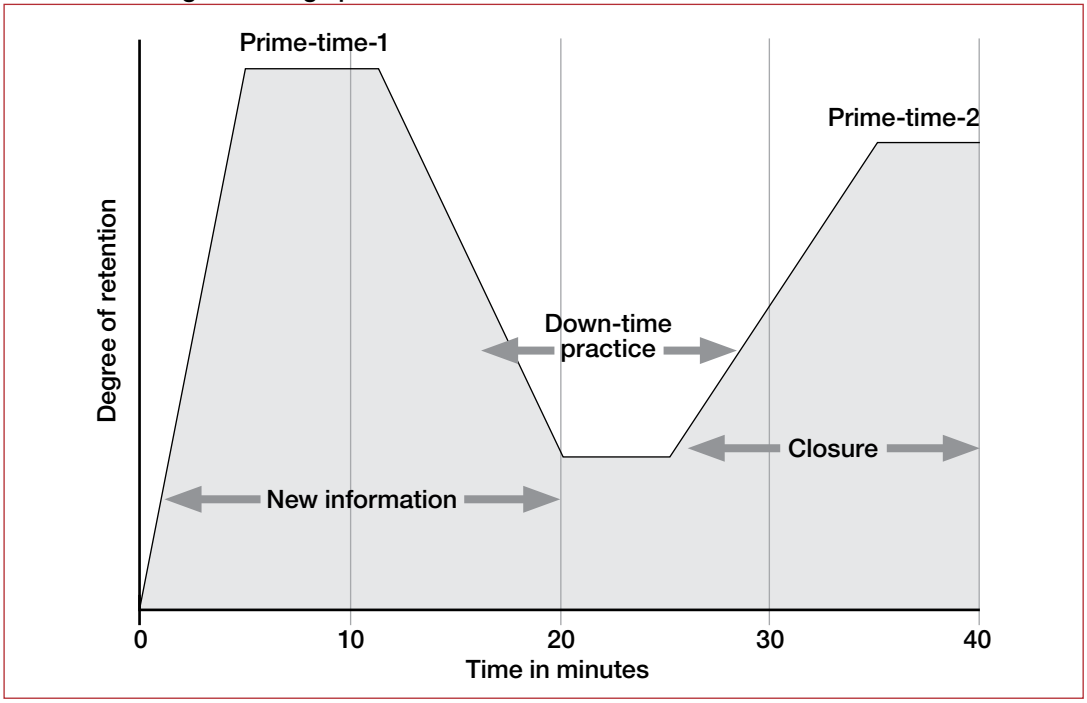
As shown in the preceding chapters, an element of external support appears to be one of the important ingredients as a catalyst for change. This chapter is written partly from the contributions of others (particularly international contributors) and also from mathematics education experts who have taken on this role with schools implementing MEP and indeed other initiatives.

The evidence from Japan is particularly striking. Here it is usual for a 'knowledgeable other', often an academic from a nearby university education department, to act as the expert, ready to give advice on research, curriculum and/or pedagogy.

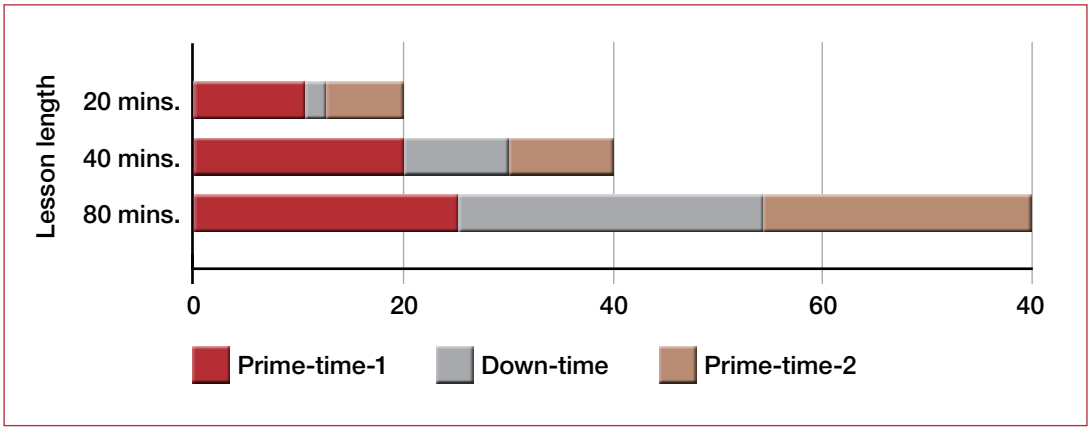
The 'external consultant' (or 'knowledgeable other') is seen as an important, indeed vital, component of the enhancement of teaching and learning. It is no surprise, for example, that academics from education departments in Japan have been at the forefront of promoting and refining open-ended approaches to introducing mathematical topics in the classroom. These academics clearly value this important practical element to their work, which commands respect amongst teachers.

Hungary has a different, more informal policy. Expert (mathematics) teachers are recognised and often based in the University Practice Schools in all the large cities. Their classrooms are always open to visitors, and staff from other schools are encouraged to observe and reflect on their lessons. One reason for this approach is that, in Hungary, most teachers of mathematics in the primary sector are effective and there has not been a need to radically change the way mathematics has traditionally been taught. They continue to use a combination of interactive whole-class teaching integrated with child-centred work. This is very much in line with the model of teaching advocated by David A Sousa[67], illustrated in the diagram below, for minimising pupil 'down-time', that is, time when a pupil is 'off-task'.

Retention during a learning episode



Approximate ratio of prime-time to down-time during learning episode



As noted in earlier chapters, CIMT has been proactive in introducing schools to Hungarian-style teaching, helping teachers to embed this into their teaching. It has taken a number of years to find effective ways of making sustainable changes to the way mathematics is taught in primary schools in the UK and, without doubt, enthusiasm alone is not enough. It is, though, an important aspect of encouraging teachers, particularly when they are expected to work outside their natural comfort zone.

It should be noted that at the start of this implementation, most UK primary teachers were used to:

- children sitting around tables, grouped according to their ability
- implementing three-part lessons for mathematics:
  - mental/oral starters (often with the children sitting on the carpet)
  - main phase
  - plenary review
- the main phase based on differentiated (group) work on the theme of the lesson.

In the Hungarian model, there is no 'carpet' work – as there is usually no carpet and no space in the classrooms for children to sit on the floor. Seating is in pairs at desks, facing the front of the classroom. This makes it easier for whole-class interactive teaching, because:

- children can easily focus on work at the front of the class
- it is easy for children to get to the board to demonstrate and articulate their solutions to the class
- the teacher can have eye contact with all children
- the teacher can easily walk between the desks to see the work of every child.

It should also be noted that the children are seated so that high-attaining pupils are paired with middle-attaining pupils and middle-attaining with low-attaining pupils. There is little grouping by similar ability or overall mixed ability: the model used is that of pupils helping each other when appropriate.

Further, rather than the three-part lessons seen in the UK, there are seven, eight or even nine related activities in a 40-45 minute lesson, with differentiation by outcome rather than activity. Many, but not all, these activities have three parts to them:

- problem introduced interactively
- pupils work on the problem(s)
- solutions reviewed interactively by the whole class under the supervision of the teacher.

Ability-group teaching does not feature widely although collaborative paired or group work on tasks and problems is encouraged and supported.

Hence, when introducing these methods to UK teachers, we were trying to radically shift their beliefs as to what could constitute effective mathematics teaching – a challenging task in which there is clearly a role for an external expert.

Rather than repeat some of the issues that have already been highlighted in earlier sections, we will present our current thinking on the role of the outside expert in effecting change in teaching mathematics in primary schools, based on experience from the many schools we have worked with.

These are our guidelines for an external expert looking to help a school to change its mathematics teaching:

1. **Initial work**
   Before starting on any work with a school, observe and reflect on its current practice in mathematics teaching. This is crucial for you as an outsider to understand what will be appropriate for enhancing mathematics teaching in the school. It also gives you a chance to get to know key teaching staff and appreciate the effectiveness of current practice in the teaching and learning of mathematics.

2. **CPD to enhance teaching**
   There is a need for the CPD (continuing professional development) which is used to enhance mathematics teaching to be non-threatening and to allow individuals to make changes at their own pace. It is important to facilitate teachers to reflect on their own practice and how it can be improved. One way of doing this is to show video clips of other styles of teaching (we have used clips from Hungary, the Czech Republic, Vietnam and China successfully) and to ask teachers to reflect on both the teaching shown and on their own teaching in the light of what they have observed. **Appendix A** (page 89) gives a proforma for this activity. The video clips we have used are on the website http://www.cimt.plymouth.ac.uk/epm

3. **Suspend belief**
As an external expert, you need to be clear about the model of teaching you are proposing for the school. You need to persuade (sometimes sceptical) teachers to suspend their belief that this will not work and to try out innovative practices that they are not familiar with and are not convinced will be effective. The use of lesson study, where colleagues work together on a jointly agreed lesson plan, observe and reflect on the lesson, is invaluable for opening teachers' eyes to the possibilities and advantages of experimenting with teaching in new and innovative ways.

4. **Demonstration lessons**
As an external expert, you should be prepared to give demonstration lessons to illustrate the model of teaching and learning you are advocating. Do this in a lesson-study format with other teachers observing; ensure that adequate time is given for review and discussion after the lesson. You will need to encourage teachers to criticise and reflect on what they have seen and what they might have done in similar circumstances.

5. **Mathematical competence**
If the mathematical competence of the teachers is an issue then CPD is needed to focus on mathematical concepts in a relevant and practical way. When implementing MEP, we often used a set of questions from the pupil resources for Years 3–6. This is given, with answers, in **Appendix B** (pages 90-96), but note that it is the mathematical thinking and strategies for solving the problems that should be emphasised, and in an interactive way.

Here is one example:

> *The product of the ages of my children is 1664. The youngest is half the age of the oldest. I am 50 years old.*
>
> *How many children do I have and what are their ages?*

Often the initial responses to this question are complete bewilderment, but usually one teacher will suggest factorising 1664. This gives

$$1664 = \underbrace{2 \times 2 \times 2 \times 2}_{16} \times \underbrace{2 \times 2 \times 2}_{8} \times 13$$

and it does not take long to realise that the only answer is '3 children whose ages are 8, 13 and 16 years'.

6. **Definitions and notation**
It is also important that teachers understand that mathematics is based on logic and rigour so that correct definitions and notations are crucial to its development.

Countdown (see website simulations at http://www.csfsoftware.co.uk/Count_info.htm) is an excellent way of improving numerical skills; it can be used at all ages. It is vitally important that the correct mathematical notation is stressed: the calculation

$7 \times 6 = 42 + 2 = 44 \times 2 = 88$

is *NOT* correct written in this way, as $7 \times 6 \neq 88$

There are no shortcuts to the written calculation: you need to write:

7 x 6 = 42

42 + 2 =44

44 x 2 = 88

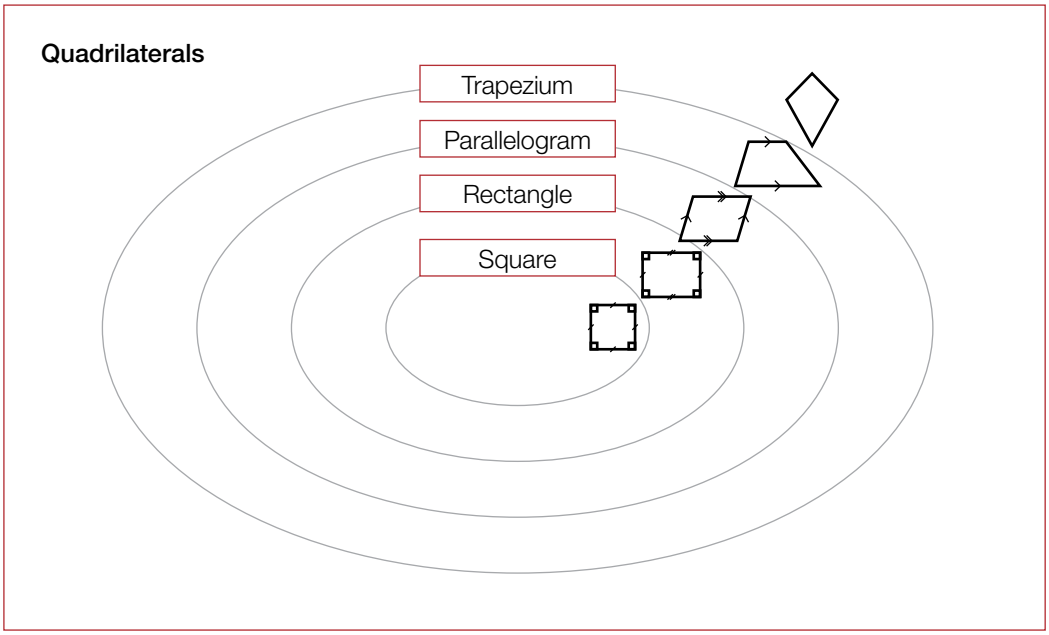
This may at first seem very inefficient but the correct use of the '=' sign is crucial for all the algebra to be encountered in a pupil's later years. Using the correct layout from the very start gives pupils a feel for the rigour of mathematics and will undoubtedly help them when they move on to working with algebraic concepts.

Another example to emphasise the precision needed in mathematics is illustrated with the following question:

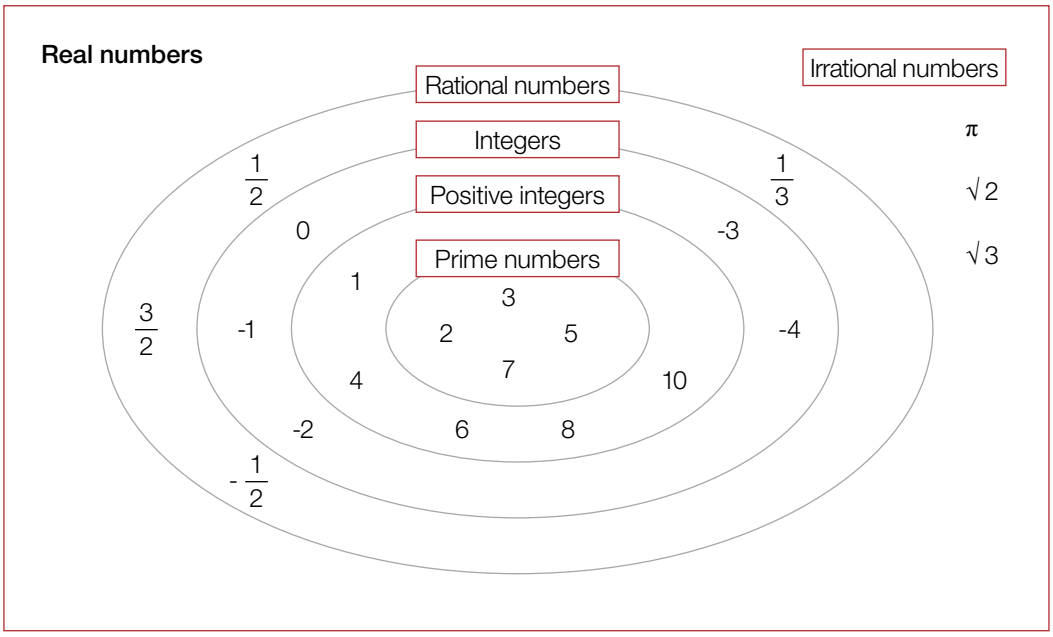
Decide whether this statement is true or false.

'All squares are rectangles.'

This is TRUE as a square meets the definition of a rectangle (a plane figure with four straight sides and four right angles). This concept can be usefully illustrated with a Venn diagram for different types of quadrilaterals.



Similarly, numbers can be classified with a Venn diagram:



Venn diagrams are very useful tools for classifying numbers and shapes, and in many other contexts; they can be used to great effect with both teachers and pupils: classifying is another important technique in mathematics.

7. Lead teacher

Identify a teacher (or teachers) who will take the lead in implementing the agreed changes. Schools must not get too dependent on the external expert but should cultivate their own expertise to oversee and troubleshoot the implementation of new teaching styles. You do, though, need to be in touch with the school's lead teacher and be ready to intervene if particular issues are threatening the overall success of implementation. Just one or two teachers with negative attitudes can have a serious impact on the overall implementation in their school.

So the role of the external expert is an important one, and in most cases is vital. If the lead teacher is sufficiently experienced, there is the possibility that they can take on this role in addition to that of the internal leadership, but our strong advice is that an external catalyst is important for successful, sustained change.

Appendix A – Proforma for reviewing video clips

(see http://www.cimt.plymouth.ac.uk/epm)

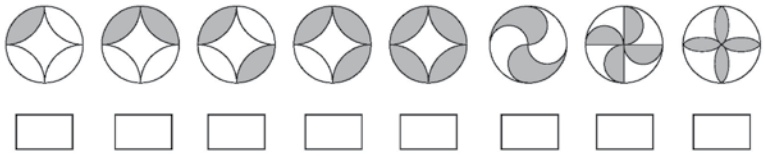
After watching the appropriate clips on the DVD (Kindergarten and Year 1 for primary teachers), respond briefly to these questions.
1. List the two or three factors in the clips that made the most impact on you.
2. What are the positive factors that you observed in the clips?
3. What are the negative factors?
4. What other issues require further discussion or information?
5. Review your own teaching in the light of the clips and consider how you might like to experiment with any of the factors or strategies. (This could be as an individual or through lesson study.)

Appendix B – Questions from Mathematics Enhancement Programme (MEP) Primary Practice Books (Y3-6)

Questions from *Mathematics Enhancement Programme (MEP)*
Primary Practice Books (Y3-6)

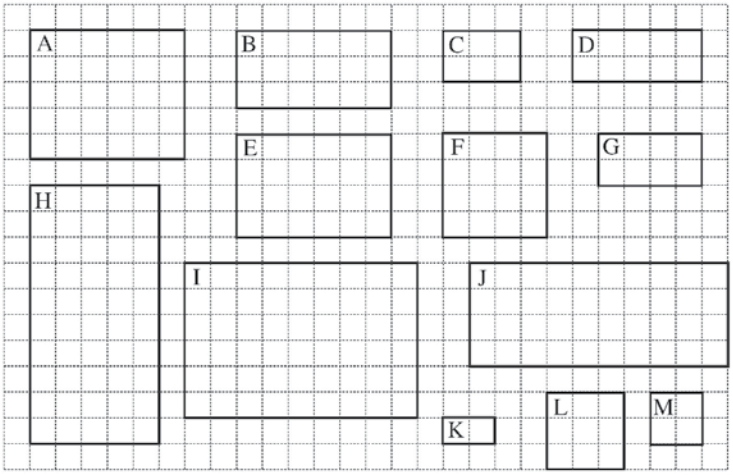
1. Write below each pattern the number of **mirror lines** it has.

a) b) c) d) e) f) g) h)



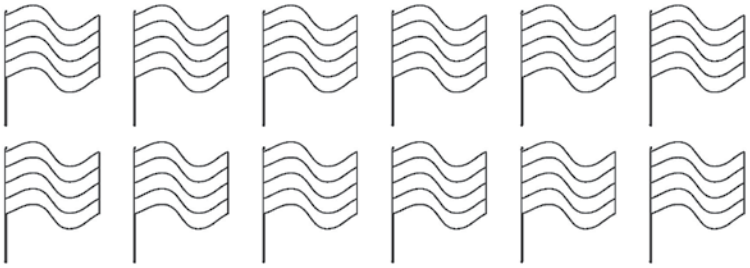
Y3b, P111, Q1

2. Colour **similar** rectangles in the same colour.



Y3b, P161, Q2

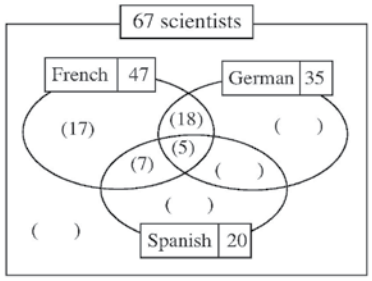
3. In how many different ways can you colour the flags *red*, *white*, *green* and *blue*? Use every colour only once in each flag. How many different ways are possible?



Y3b, P167, Q1

CIMT, Plymouth University (www.cimt.plymouth.ac.uk)

4. Among 67 scientists at a conference,
47 speak French,
35 speak German,
20 speak Spanish,
12 speak French and Spanish,
11 speak German and Spanish,
5 speak all three languages.



a) Complete the *Venn* diagram.

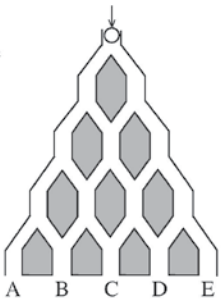
b) How many scientists speak:
i) only French ☐ ii) only German ☐ iii) only Spanish? ☐

c) How many scientists speak Spanish and German but not French? ☐

d) How many scientists speak neither Spanish nor German nor French? ☐

Y4b, P151, Q3

5. A marble is dropped into this maze and has an equal chance of falling to the left or to the right.



a) In how many ways can the marble come out at:
A .... B .... C .... D .... E ....?

b) Where is the marble most likely to come out? ....

c) Write the ratio of the chance of where it comes out.

A	B	C	D	E
☐ :	☐ :	☐ :	☐ :	☐

Y4b, P156, Q3

6. The product of the ages of my children is 1664. The youngest is half the age of the oldest. I am 50 years old.
How many children do I have and what are their ages? .....

Y4b, P172, Q3

7. In a **magic square**, the sum of the numbers in each row, column and diagonal is the same. Complete these magic squares.

a)

	11	7
9		
	5	10

b)

10	3	
5		
	11	4

c)

14		12
10		8

Y5b, P167, Q4

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8. Use each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 **only once** to make **five** whole numbers, so that one number is twice, another number is three times, another number is four times and the last number is five times the smallest number.

Y5b, P167, Q3

9. Decide whether the statements are *true* or *false*. Write T or F in the boxes.

a) All squares are rectangles. ☐

b) All squares are parallelograms. ☐

c) The diagonals of any parallelogram are not equal in length. ☐

d) Every parallelogram which has perpendicular diagonals is a square. ☐

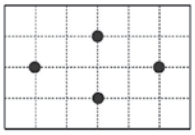
e) Not every parallelogram with equal sides is a square. ☐

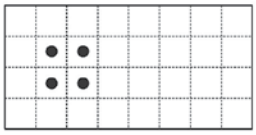
f) A parallelogram with equal sides and equal angles is a square. ☐

Y5b, P165, Q1

10. Each diagram is the map of a field in which there are 4 wells. Show how the field could be divided into 4 **congruent** parts so that each part has exactly one well.

Use only the marked grid lines for your divisions.

a) 

b) 

Y5b, P172, Q5

11. *Freddy Fox* decided that from that day forward he would always tell the truth on Mondays, Wednesdays and Fridays but he would always tell lies on the other days of the week

One day he said, '*Tomorrow I will tell the truth.*'

On which day of the week do you think he said this? ....

Y5b, P174, Q2

12. Use each of the natural numbers from 1 to 16 only **once** to form 8 pairs of numbers so that the sum of each pair is a **square** number.

For example, (2, 14) is a possible pair, as $2 + 14 = \underline{16} = 4 \times 4$

(,), (,), (,), (,), (,), (,), (,), (,)

Y5b, P175, Q1

13. If the statement is true, write 'T' in the box and if it is false, write 'F'.

a) Every **isosceles triangle** has angles of 60°. ☐

b) No **rectangle** has adjacent equal sides. ☐

c) The **diameter** of a circle is twice the length of its radius. ☐

d) The **circumference** of a circle is its radius multiplied by π. ☐

e) There is a **prism** which has congruent faces. ☐

f) A square-based **pyramid** has 5 vertices, 5 faces and 8 edges. ☐

g) If the diagonals of a quadrilateral **bisect** each other at right angles, the quadrilateral is a **rhombus**. ☐

h) A **tangent** to a circle can touch the circle at more than 1 point. ☐

Y6b, P155, Q1

14. Decide whether each statement is true or false and write T or F in the box.

a) The **product** of two numbers can be less than each of the two numbers. ☐

b) The **arithmetic mean** of two negative numbers can be positive. ☐

c) There is an **isosceles** triangle which has two right angles. ☐

d) There is a positive fraction less than 1 which is equal to its **reciprocal.** ☐

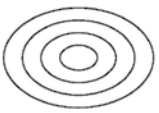
e) If a **product** is zero, at least one of its factors is zero. ☐

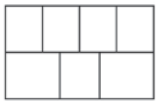
f) If the areas of two triangles are equal, the triangles are **congruent**. ☐

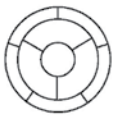
g) There is a quadrilateral which is both a **deltoid** and a **parallelogram** but is not a square. ☐

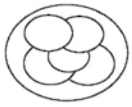
Y6b, P161, Q1

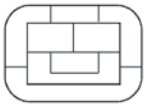
15. Colour these maps using the smallest number of different colours possible. Adjacent countries must not have the same colour but countries with the same colour can meet at a point.

a) 

b) 

c) 

d) 

e) 

f) 

Y6b, P170, Q3

16. If x and y are positive integers, find as many values as possible for x and y which make this equation true.

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{8}$$

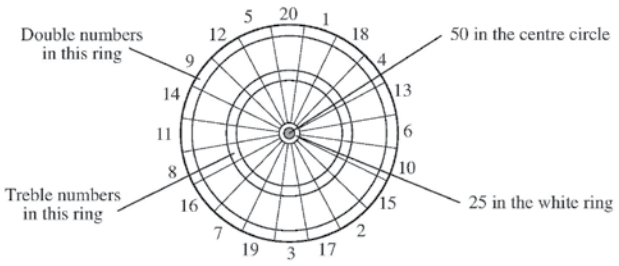
Y6b, P170, Q4

17. We have 30 silver coins. Although they all look the same, we know that one of the coins is fake and is lighter than the others.

If we tried to find out which coin is fake using a 2-pan balance, what is the **least** number of weighings we would need to do?

Y6b, P174, Q6

18. This is a diagram of a *dart board*.



a) What different scores between 50 and 60 can you get with one dart?

b) If you throw 3 darts one after the other and all of them score, what is the:

i) **highest** score possible ii) **lowest** score possible?

c) Using 1, 2 or 3 darts, what is the lowest score that is **impossible** to get?

Y6b, P175, Q1

Questions from *Mathematics Enhancement Programme (MEP)*
Primary Practice Books (Y3-6)
ANSWERS

1. 1, 1 2, 1, 4, 0, 0, 4 — Y3b, P111, Q1

2. B, G, H and K are similar; C, I and E are similar;
L, M and F are similar; D and J are similar. — Y3b, P161, Q2

3. $4 \times 3 \times 2 \times 1 = 24$ — Y3b, P167, Q1

4. b) i) 17 ii) 6 iii) 2 c) 6 d) 6 — Y4b, P151, Q3

5. a) A 1 B 4 C 6 D 4 E 1 b) C
c) A : 1; B : 4; C : 6; D : 4; E : 1 — Y4b, P156, Q3

6. 8, 13 and 16 — Y4b, P172, Q3

7. a)

6	11	7
9	8	7
9	5	10

(24)

b)

10	3	8
5	7	9
6	11	4

(21)

c)

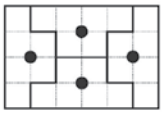
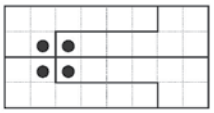
14	7	12
9	11	13
10	15	8

(33)

Y5b, P167, Q4

8. 18, 36, 54, 72 and 90 — Y5b, P167, Q3

9. a) T b) T c) F d) F e) T f) T — Y5b, P165, Q1

10. a)  b)  — Y5b, P172, Q5

11. Saturday — Y5b, P174, Q2

12. (9, 16), (11, 14), (12, 13), (10, 15) - all add to 25
(2, 7), (5, 4), (3, 6), (1, 8) - all add to 9 — Y5b, P175, Q1

13. a) F b) F c) T d) F e) T f) T g) T h) F — Y6b, P155, Q1

14. a) T b) F c) F d) F e) T f) F g) T — Y6b, P161, Q1

15. a) 2 b) 3 c) 4 d) 3 e) 4 f) 4 — Y6b, P170, Q3

16.

x	16	12	24	10	40	9	72
y	16	24	12	40	10	72	9

So there are 7 pairs of values but note that three are repeated, so the answer 4 would also be valid. — Y6b, P170, Q4

CIMT, Plymouth University (www.cimt.plymouth.ac.uk)

17. 1) Divide the 30 coins into 3 groups of 10.

    Weigh Group 1 against Group 2. If they balance, the fake coin must be in Group 3. If they do *not* balance, the fake coin is in the lighter group.

    2) Divide the 10 coins in the lightest group into 3 groups (3, 4, 4).

    Weigh the two groups of 4. If they balance, the fake coin must be in the group of 3. If they do *not* balance, the fake coin must be in the lighter group of 4.

    3) If the fake coin is in the group of 3, weigh one coin against another coin. If they balance, the 3rd coin is fake. If they do *not* balance, the lighter coin is the fake.

    or If the fake coin is in a group of 4, weigh 2 coins against the other 2 coins. The fake coin is in the lighter pair.

    4) Then weigh the coins in the lighter pair against each other. The fake coin is the lighter of the two.

    Answer: To find the fake coin we would need to do at least 3 weighings and at most 4 weighings. Y6b, P174, Q6

18. a) 51, 54 and 57

    b) i) 180 ii) 3

    c) 163 Y6b, P175, Q1

7 Recommendations for good practice

Professor David Burghes and Jodie Hunter,
CIMT, Faculty of Health, Education and Society, Plymouth University

In this chapter we summarise our four key recommendations for good practice.

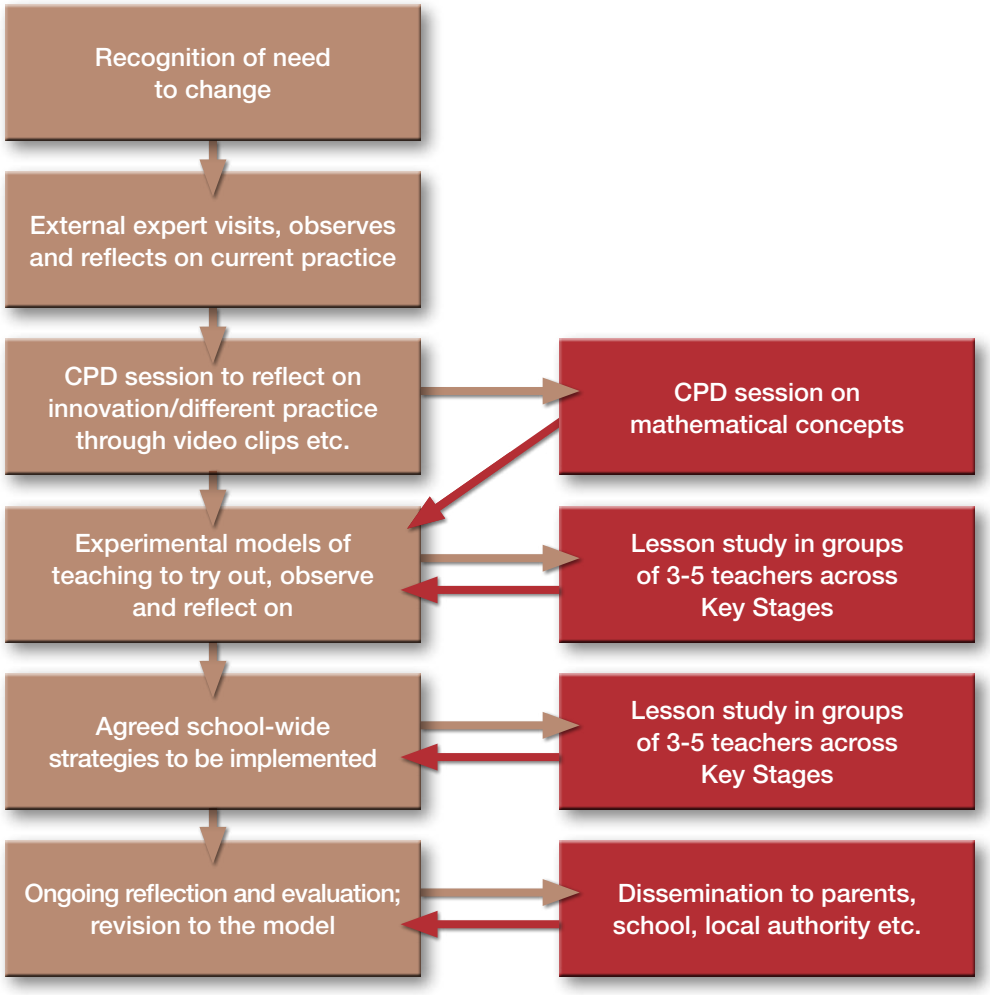
1. **Overall change model**

   In the previous chapters, we have detailed the role of an external expert as a catalyst for change. Here, in this final chapter, we give our recommendations for effective and substantial change in the teaching and learning of mathematics. These are mainly based on our experience with implementing MEP Primary Mathematics but we are convinced that this change model has much wider applicability.

   Following this model does not guarantee success in enhancing mathematics teaching and learning but we contend that it will at least improve the chance of success. Whilst it is based on our experimental evidence of successful change in mathematics teaching and learning, we recognise that there are numerous other change models that might work for you. We know that this model, outlined below, has worked well in a number of primary schools for innovating and sustaining enhancement in mathematics teaching and learning.

   The change model is summarised in the diagram below:

   Change model



2. **Lesson study**

Each of the stages identified here is important, but, for sustainability of progress, the use of lesson study is crucial. Here we recommend the Japanese version of lesson study in which (also see Section 1.2 by Masataka Koyama and Chapter 4 by Derek Robinson) teachers of mathematics first set their overarching aim for lesson study. This might be as straightforward as

> 'We aim to enhance the teaching and learning of mathematics.'

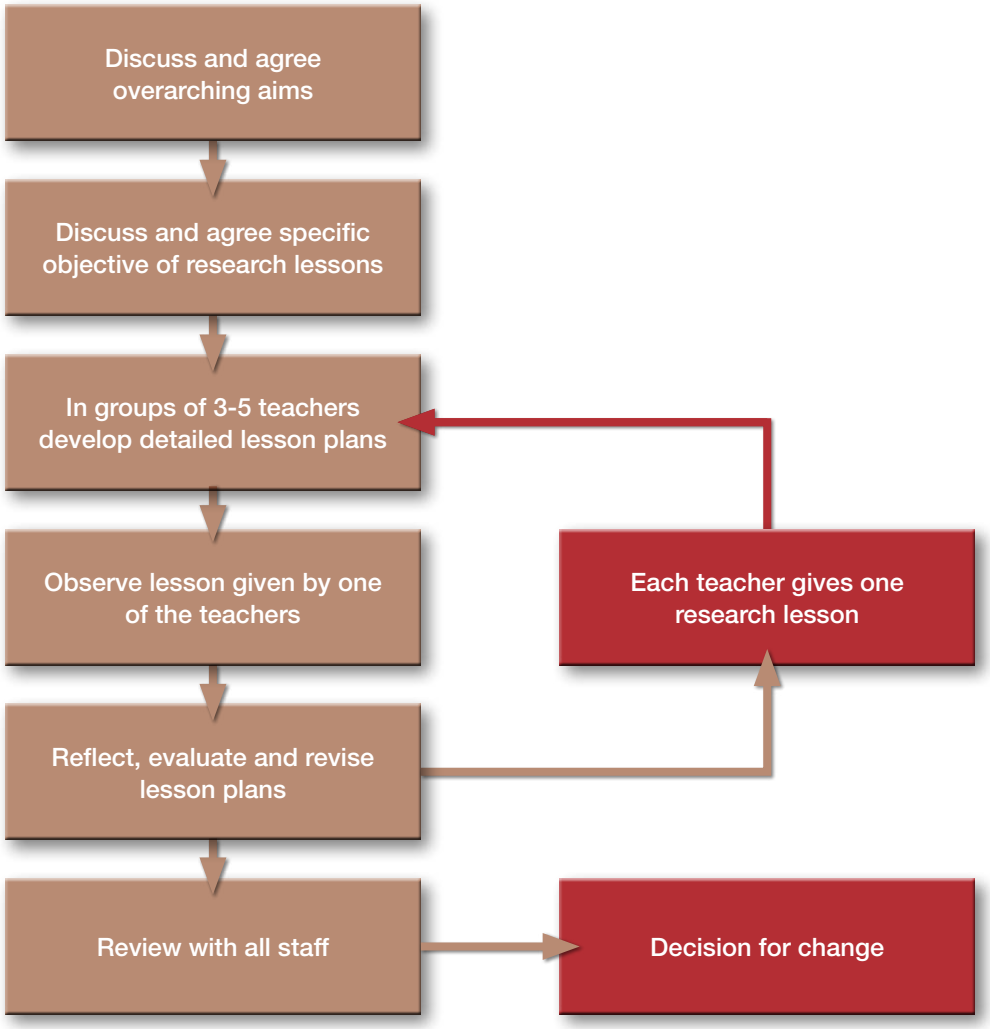
The next step is for them to agree amongst themselves, ideally with the help of an external expert, the specific objectives to be met in the research lessons. For example:

- *We want our pupils to enjoy their mathematics.*
- *We want pupils to articulate and demonstrate their solutions to the class.*
- *We want our pupils to have the confidence to work independently on problems in mathematics.*
- *We want pupils to improve their understanding of the logic and rigour needed for mathematics.*

In groups of three or four, teachers then plan a research lesson that meets at least two of the objectives. The lesson is taught, observed by the other members of the group, after which an in-depth review is carried out during time set aside specifically for this purpose. Action plans are noted for wider dissemination and for use in the next cycle of lesson study. The next research lesson will be given by another member of the group, with the same process of jointly planning and reviewing.

This important process is summarised in the diagram below:

Lesson study model



3. Mathematics leadership

Lesson study is one of the key points for sustainability – but of equal importance is the role of a mathematics co-ordinator in the school to undertake the leadership of the initiative to enhance mathematics teaching and learning. The co-ordinator needs to be available, particularly in the initial stages, to:

- encourage, support and listen to colleagues
- use staff meetings to discuss progress and review lesson study
- intervene when there are problems and uncertainties that are causing concern to teachers.

In summary, the mathematics co-ordinator should maintain a leadership role which is flexible, tolerant and receptive to necessary changes to ensure the overall success of the initiative.

The ongoing UK government initiative to train specialist mathematics teachers fits well with their role in schools to enhance mathematics teaching and learning. We have learned over the past decade that a straightforward cascade model does not necessarily lead to successful change; however with suitable leadership, lesson study is a method whereby teachers can benefit from their own observations and group discussion, giving sustainable improvements in teaching and learning.

4. Mathematical competence

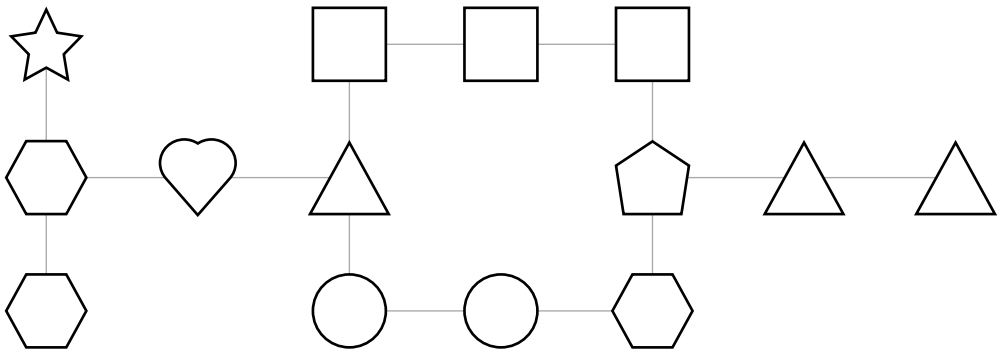
In Chapter 6 we highlighted the issue of the mathematical competence of teachers. This is a crucial aspect of success in improving mathematics in your school.

From international research studies it is evident that countries which achieve well mathematically begin by developing strong mathematical foundations in the primary phase of education. They encourage and enable their pupils to think mathematically and be creative and confident in using mathematics from the start of their education. This results in students entering employment confident in using and applying their mathematical knowledge.

To try to illustrate what we mean by 'thinking mathematically', we will give some examples that are designed for pupils but can be used for CPD with teachers, so that they can begin to understand the concept.

Example 1 (Year 1 resource)

Find three numbers which when added together equal the number 9. Do not use 0. Each group of three shapes below equals 9 and the same shape stands for the same number. Can you work out what shape is what number?



Comments on Example 1

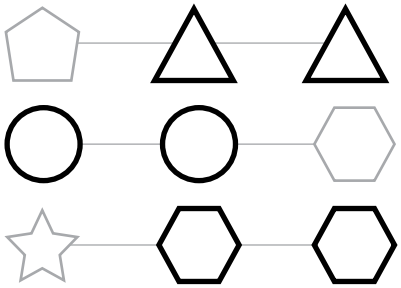
The obvious method is to use trial and error, but this is not how it would be tackled in Hungary. Clearly the square will be 3. Now consider lines when there are just two equal numbers; these are:

1, 1, 7

2, 2, 5

4, 4, 1

Look now at the shapes. There are also three lines with just two equal shapes; these are:



To know which shape line is which set of numbers, note that the **hexagon** occurs **three times** in the shape lines (once in line 1 and twice in Line 3). Now look at the numbers, the only number that occurs three times is 1. So the shape rows have to be (in the corresponding order):

5, 2, 2

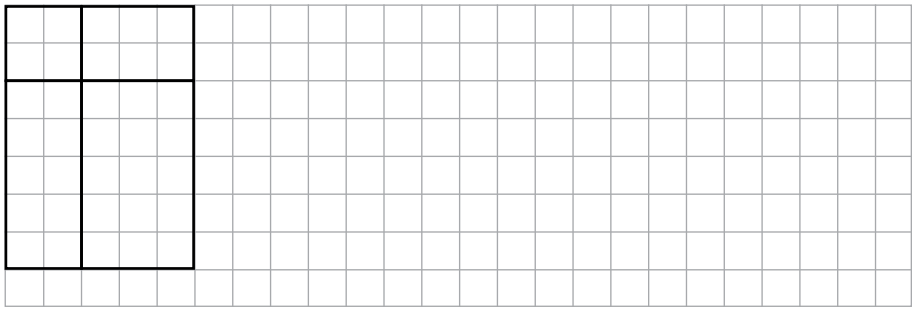
4, 4, 1

7, 1, 1

The rest of the solution now follows: using Hexagon = 1 and Triangle = 2, we can see from the original diagram that **Heart = 6**.

Example 2 (Year 2 resource)

How many rectangles can you see on this figure? Draw each of them again on the grid. Colour the squares blue.

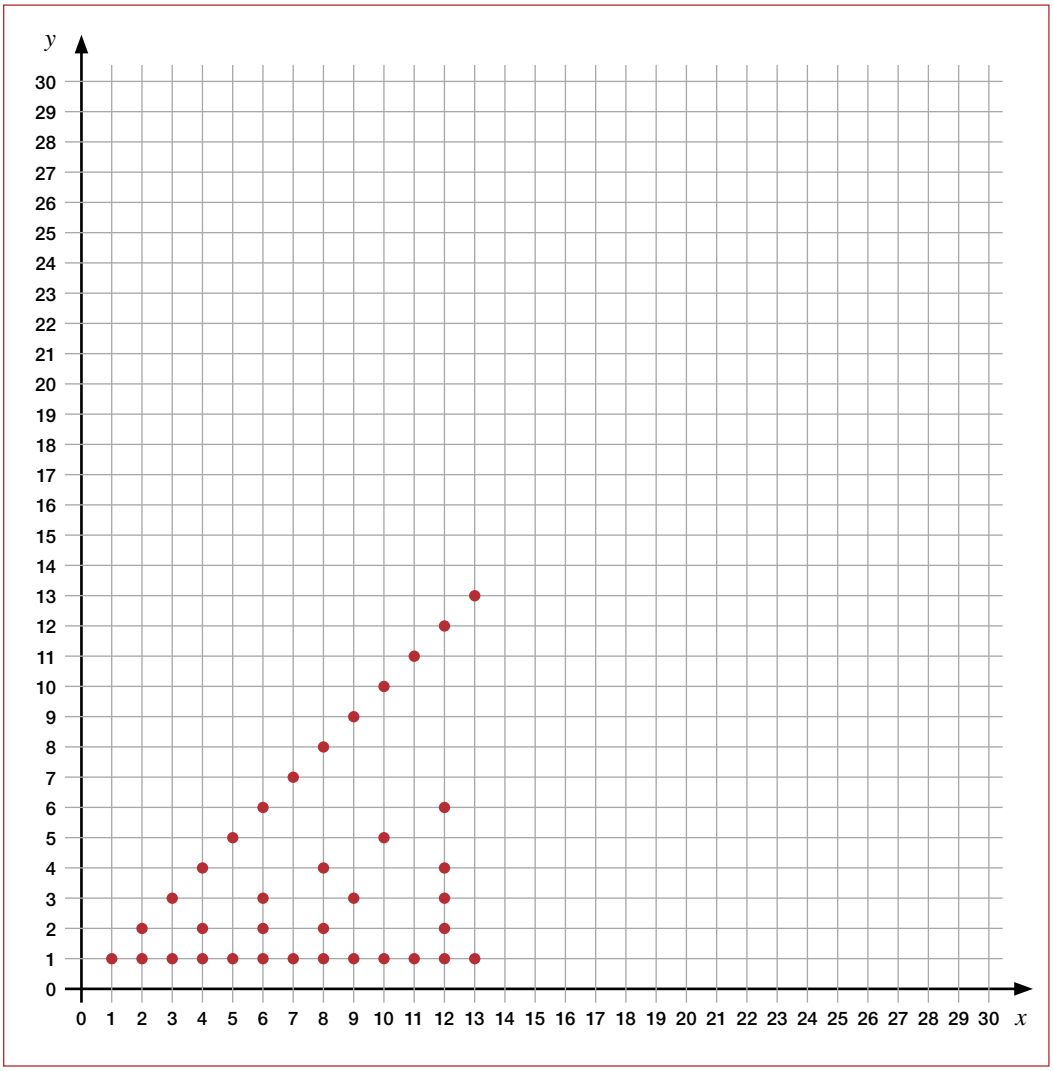


Comments on Example 2

This looks straightforward and it is easy to see the four rectangles (one is a square) that make up the shape. The next stage is to identify the four rectangles made from combining two of the first rectangles. Then the real mathematical thinker will also identify the whole rectangle as being the ninth that can be seen in this diagram.

Example 3 (Year 5 resource)

a Continue drawing the dots. y is a factor of x and $x \leq 30$.



b) Complete these statements.

i) x is a _____ of y.

ii) A = {has exactly two factors} = { }

iii) B = {has an odd number of factors} = { }

iv) C = {has only one factor} = { }

Comments on Example 3

This is an illuminating way of identifying the concepts of:

- *prime numbers*
- *square numbers*
- *multiples*

as well as showing how patterns can be developed and generalised.

Algebra

Historically within Western schooling, algebra has been situated as a gate-keeper – reserved for higher-achieving students and introduced independently in secondary school after a primary curriculum with a strong emphasis on computation and arithmetic. Many primary teachers themselves are reluctant to teach algebra due to past negative experiences. However, international research advocates the introduction of algebra from primary school to ensure a sound foundation in mathematics to be built on in later years. In countries which perform at high levels mathematically, algebra is introduced in the primary sector.

We have found it useful first to clarify notation; for example, indices and the use of expressions such as $(2x^2)^2$, which is in fact,

$$(2x^2) \times (2x^2) = (2 \times x \times x) \times (2 \times x \times x)$$
$$= 2 \times 2 \times x \times x \times x \times x$$
$$= 4x^4, \textit{ etc.}$$

Check this out with numbers, and stress that algebra, rather than being a difficult topic, is entirely straightforward once you know the definitions and rules.

One way to illustrate the power of algebra is to use what at first sight appears to be a trick, which can be understood when the algebraic version is studied.

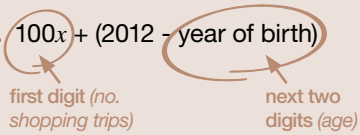
Your Age Through Going Shopping

1. How many times in a week would you like to go shopping? It has to be more than once and less than eight!
2. Multiply this number by 2
3. Add 5
4. Multiply by 50
5. Add 1762 if you have not yet had your birthday this year and 1763 if you have already had your birthday
6. Subtract the four digits of the year you were born
7. You should now have a three-digit number
   - The first digit is the number of times you wanted to go shopping
   - The next two digits are your age!

Of course, the subject of this trick can be changed to suit the audience. It will always amaze but it is the use of algebra that gives the game away.

Here is the sequence of operations, completed in algebraic format:

1. Let x = number of times in a week that you would like to go shopping
2. $2x$
3. $2x + 5$
4. $50 \times (2x + 5) = 100x + 250$
5. $100x + 250 + 1762$ (if not yet had birthday)
6. $100x + 2012$ - (year of birth)
7. $100x$ + (2012 - year of birth)



This works for the year 2012, but '1763' and '1764' should be used in the procedure for the years 2013, etc.

Conclusion

We hope that you have enjoyed reading accounts of our experiences in enhancing mathematics teaching and learning in primary schools. We recognise that many people struggle with mathematics from an early age right through to adulthood. This does not need to be the case and, alongside the changes in teaching strategies highlighted in this publication, we need to state the obvious: that is, that without a sound grasp of number, what could be termed 'number sense', pupil progress will always be a problem. In mathematically high-performing countries, we note that such basics as *number bonds* and *multiplication tables* are learned in the first two or three years of primary instruction and remembered for life.

We believe passionately that putting in place a strong and rigorous mathematical foundation in primary schools is the long-term solution for any country, not just the UK, to produce young people (and teachers) who are capable of using and applying their mathematical prowess with confidence and, in our view just as importantly, enjoying and appreciating mathematical thinking throughout their education.

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CfBT Education Trust
60 Queens Road
Reading
Berkshire
RG1 4BS

+44 (0) 118 902 1000

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PMS 5970 06/12/R ISBN 978-1-907496-75-2